



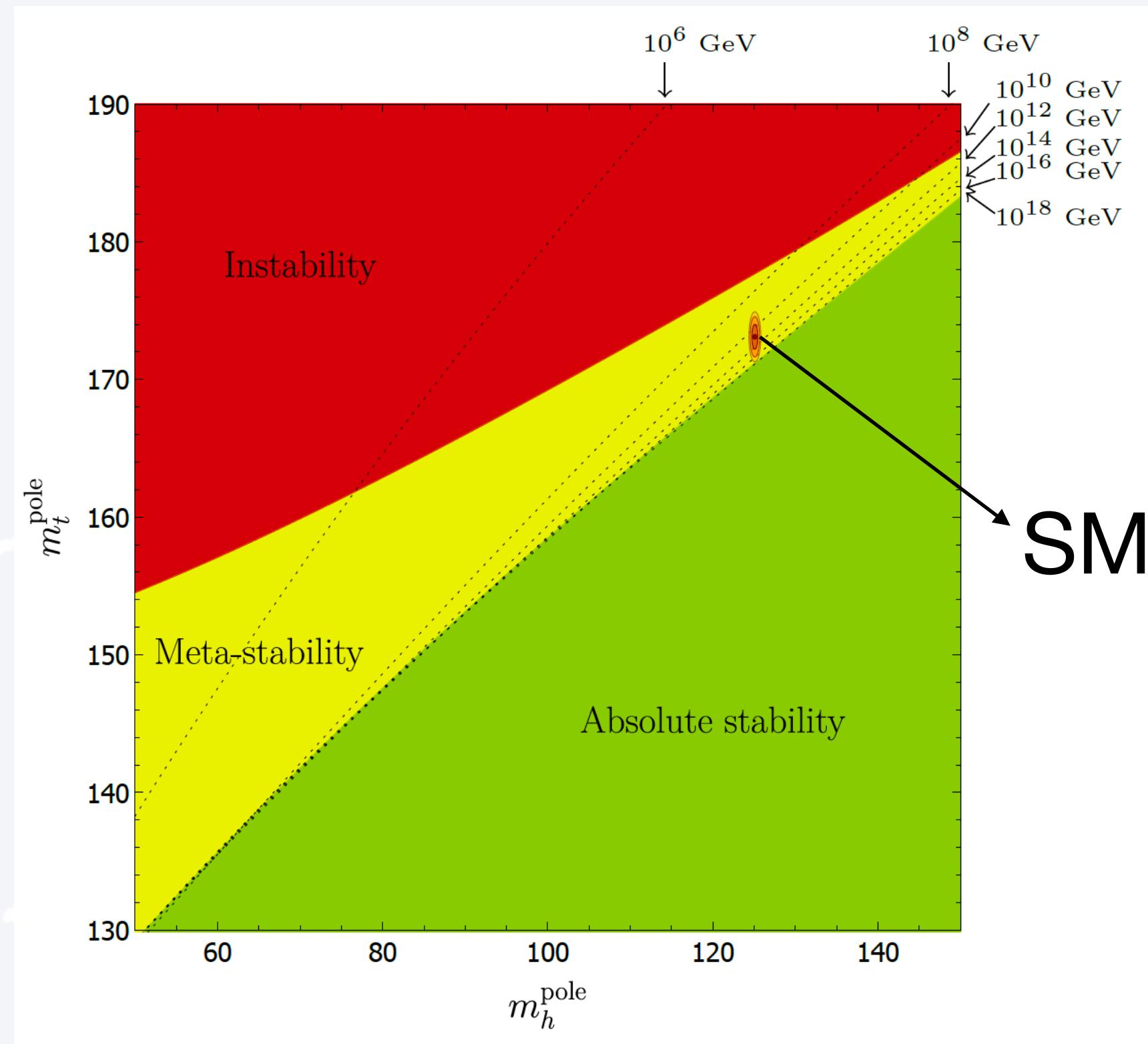
Calculational challenges for high-energy colliders

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Université catholique de Louvain
Università di Bologna

Where do we stand?

$$\mathcal{L}_{SM}^{(4)} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}i\cancel{D}\psi + (y_{ij}\bar{\psi}_L^i\phi\psi_R^j + h.c.) + |D_\mu\phi|^2 - V(\phi)$$



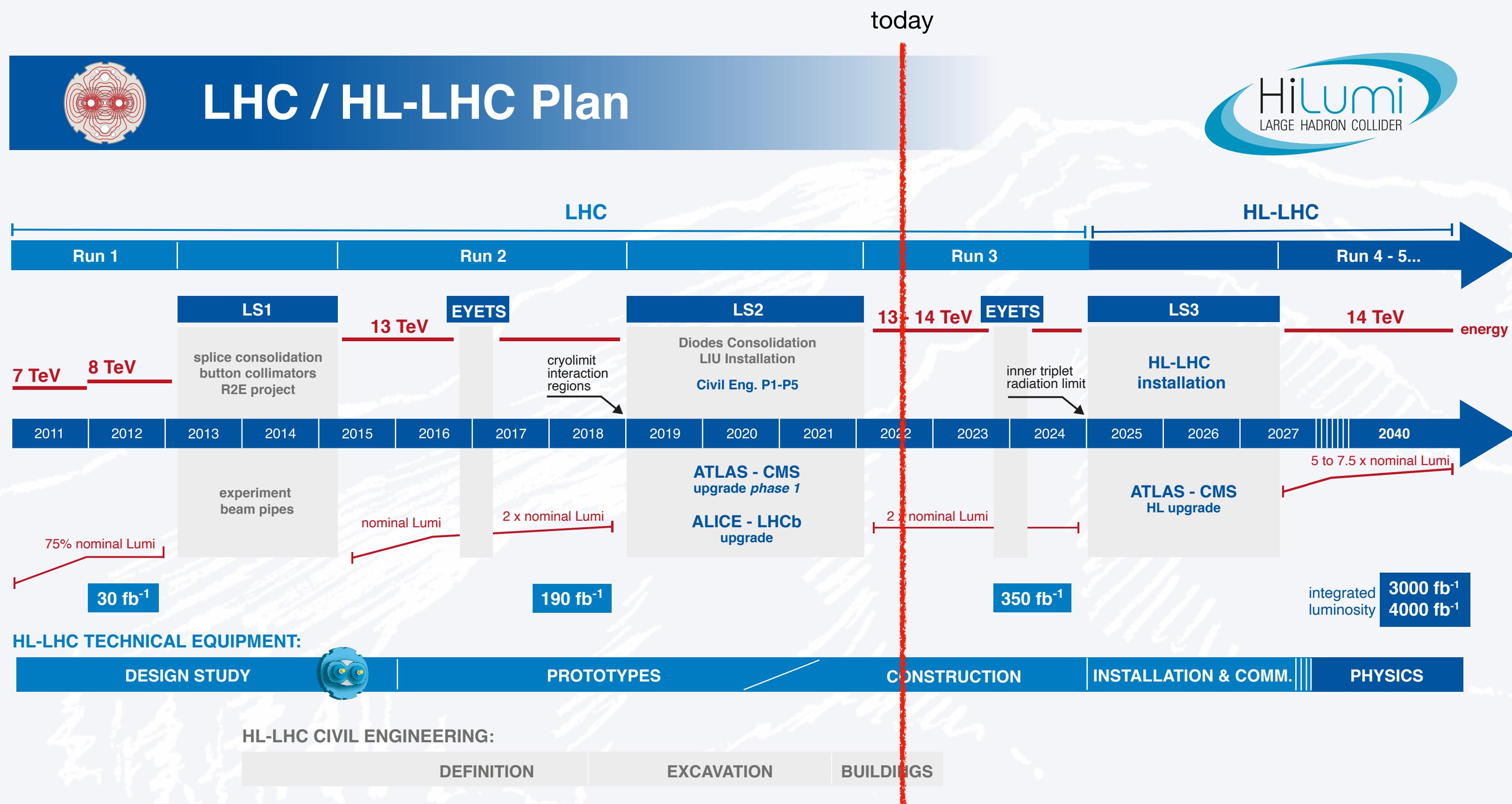
- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetries.
- Matter is organised in chiral multiplets of the fund. representation.
- The $SU(2) \times U(1)$ symmetry is spontaneously broken to $U(1)_{EM}$.
- Yukawa interactions lead to fermion masses, mixing and CP violation.
- Matter+gauge group \Rightarrow Anomaly free
- Renormalisable = valid to “arbitrary” high scales.
- A number of accidental symmetries seen in Nature.
- Neutrino masses can be accommodated in two distinct ways.

CC#1: Determine the SM parameters in terms of underlying UV dynamics, and in particular the Higgs mass is out of control.



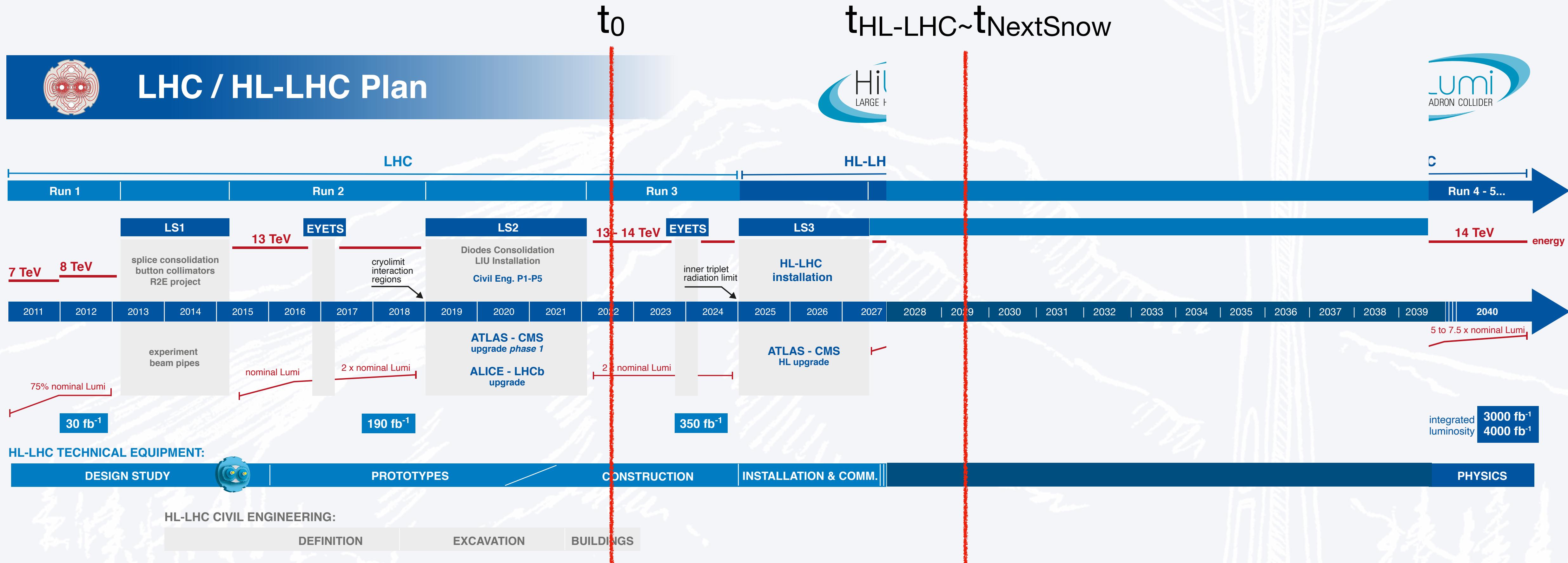
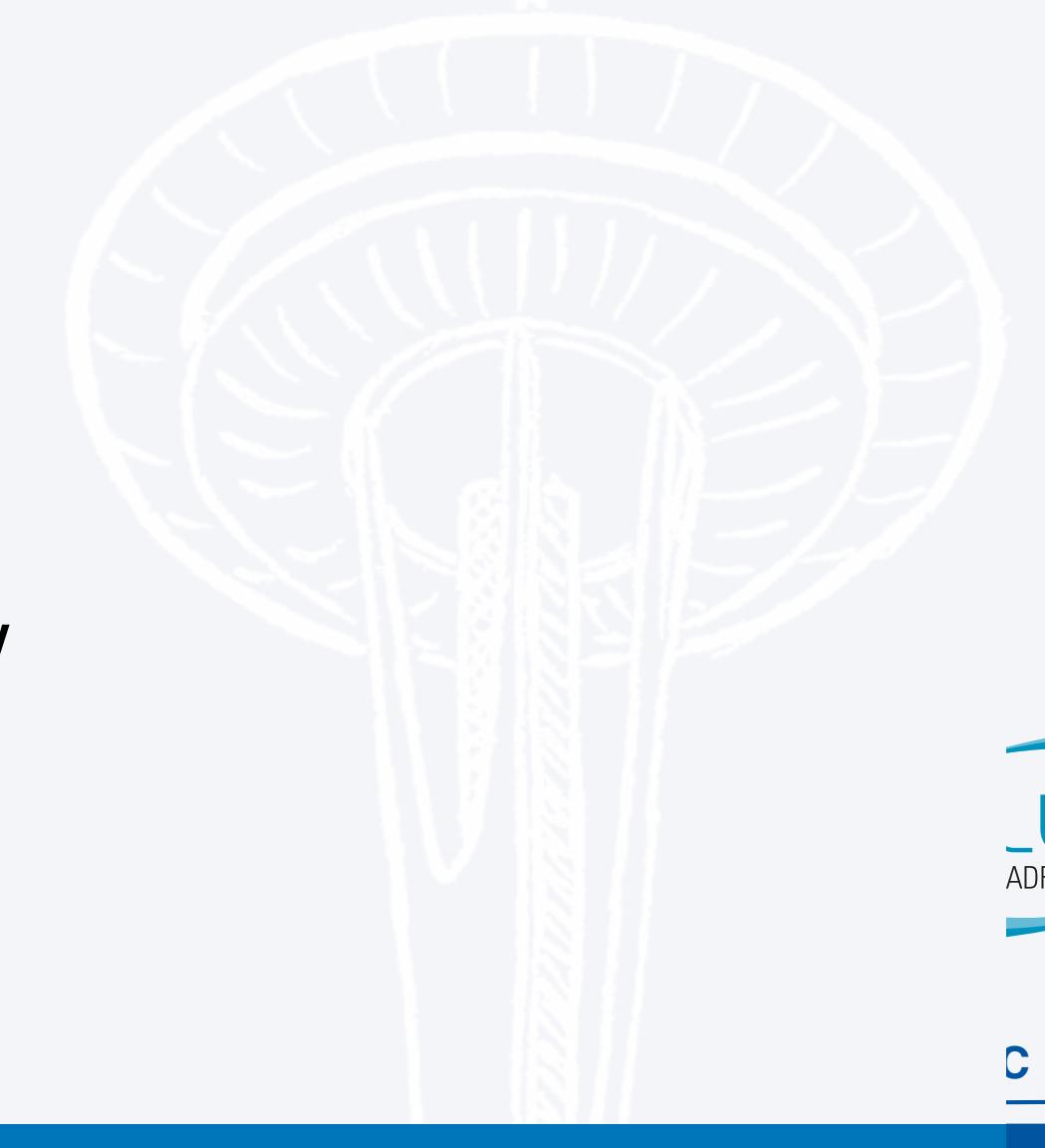
Where do we stand?

The LHC reference frame and unit of time



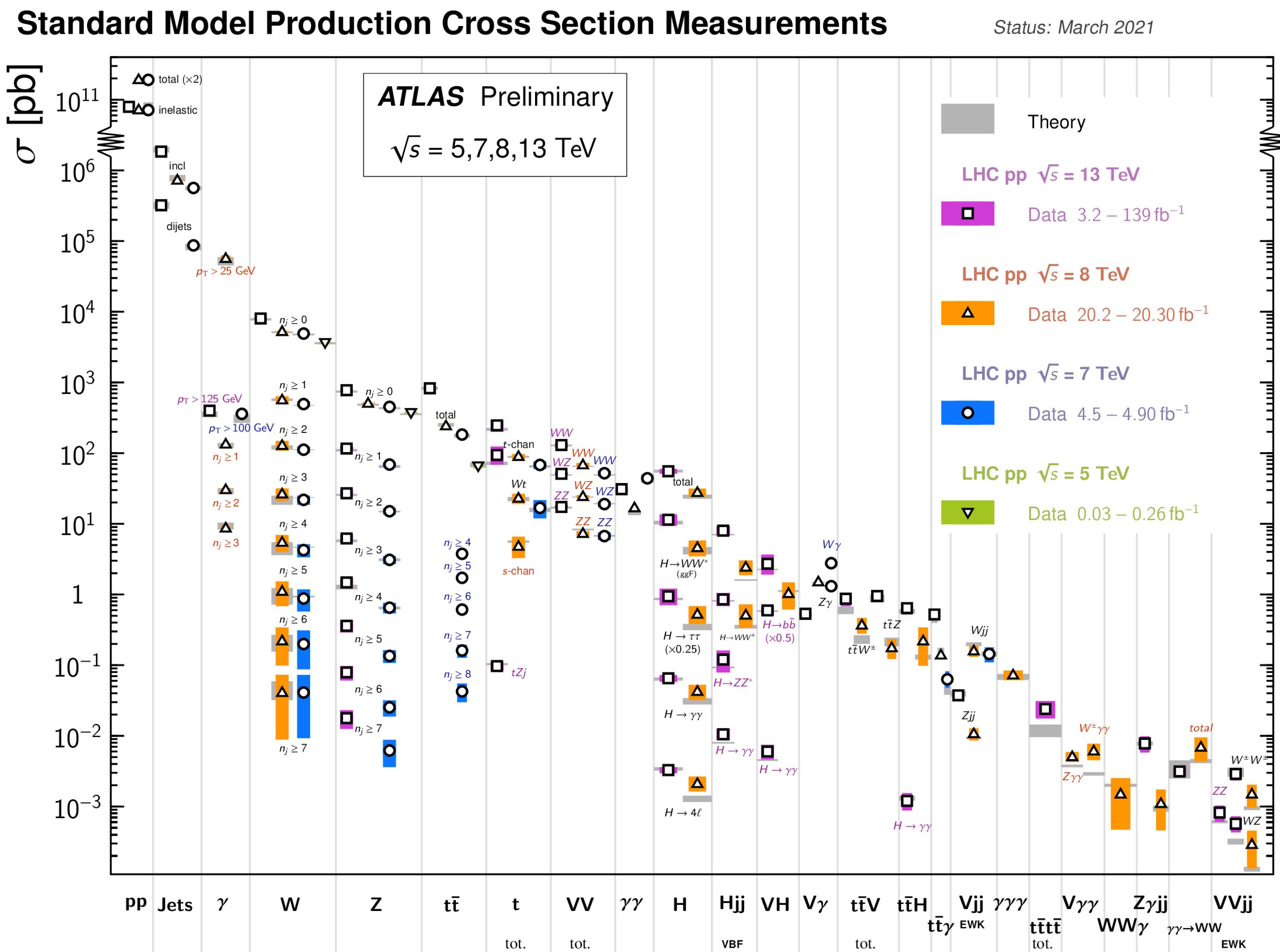
Where do we stand?

The LHC reference frame and unit of time

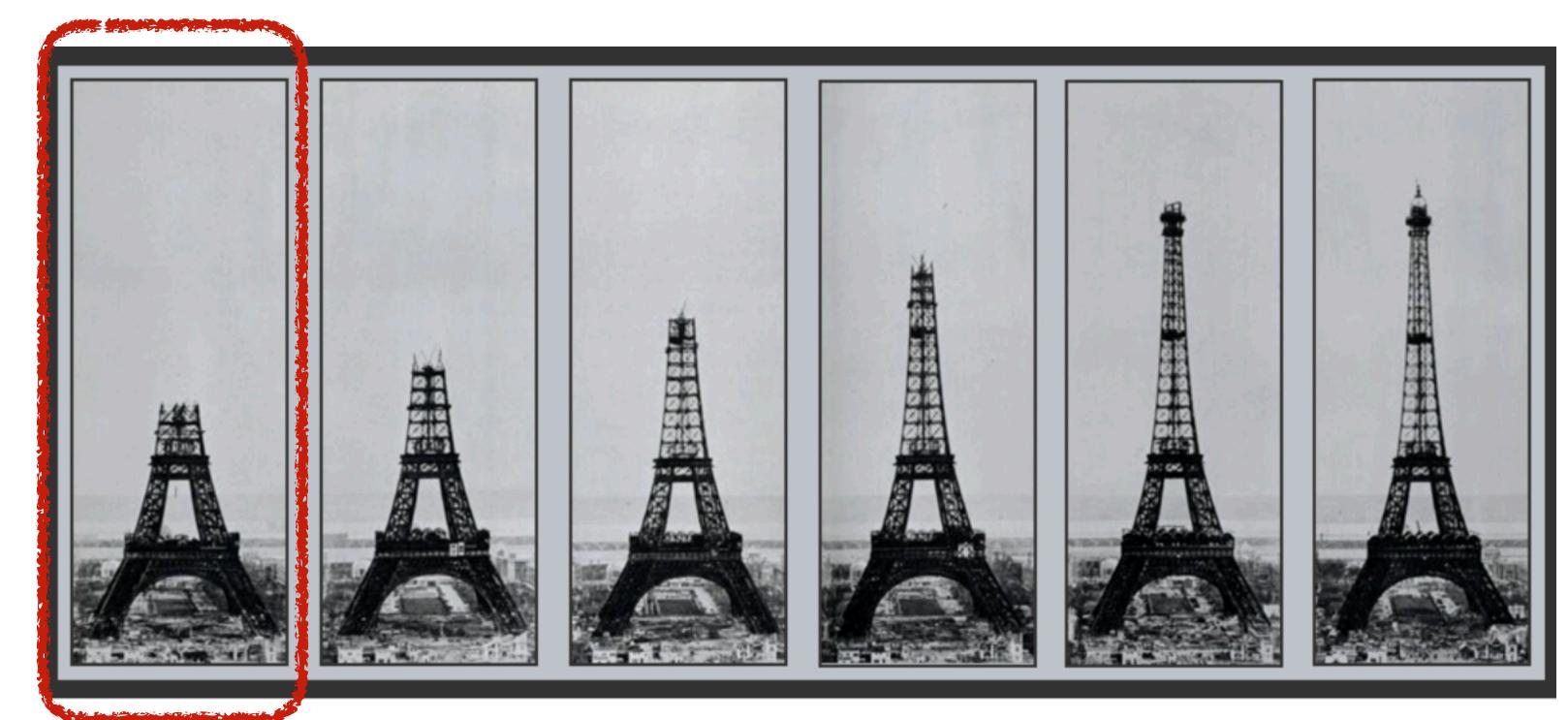


We are at 1/3 of our adventure with 1/20 of the expected data

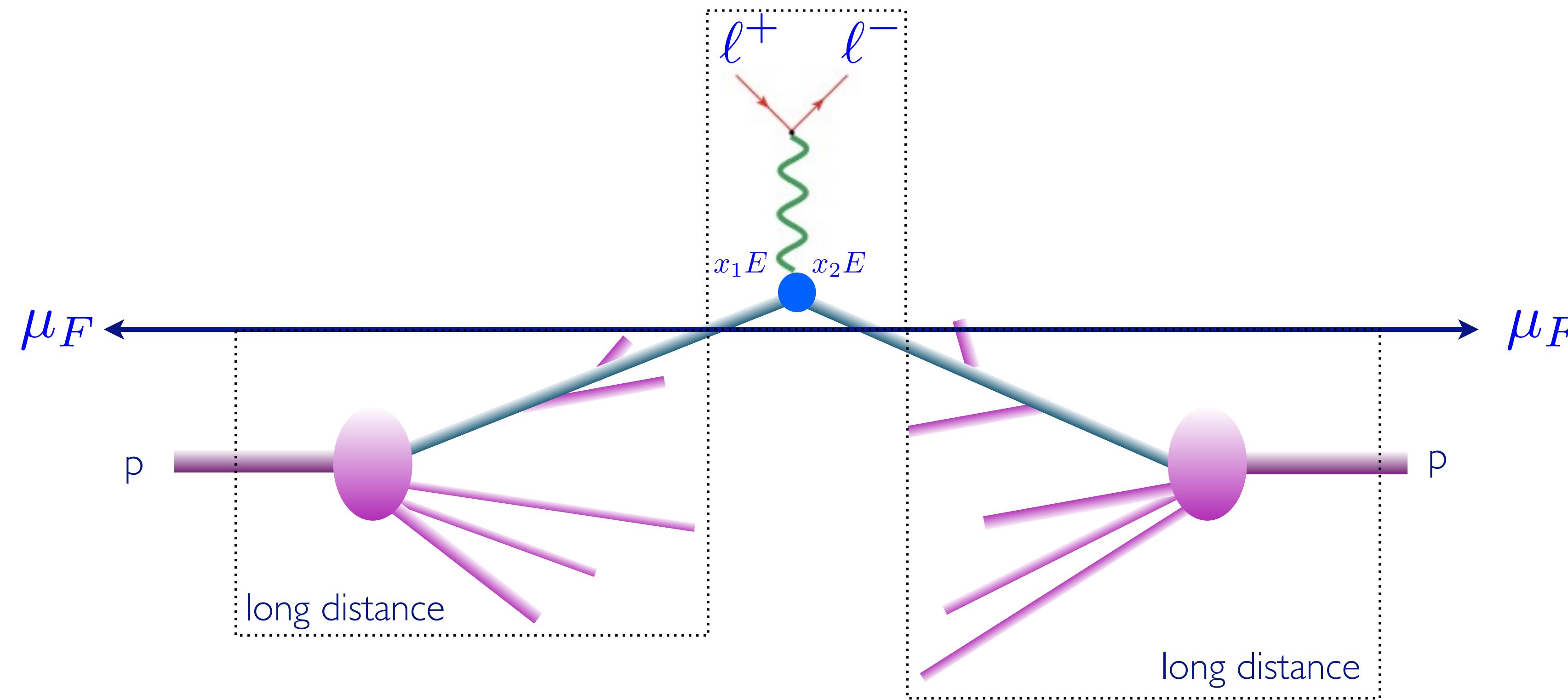
Where do we stand?



- Tangible results of an amazing experimental effort over a 10+ year span, accessing a wide range of final states, each with very different challenges.
 - Theory predictions seem adequate. (The key role of MCs is hidden in this plot).
 - Comparison with SM predictions shows that we have the necessary theoretical and experimental control to move onto the next phase.



LHC master formula



$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \rightarrow X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

Measure (& evolve perturbatively)

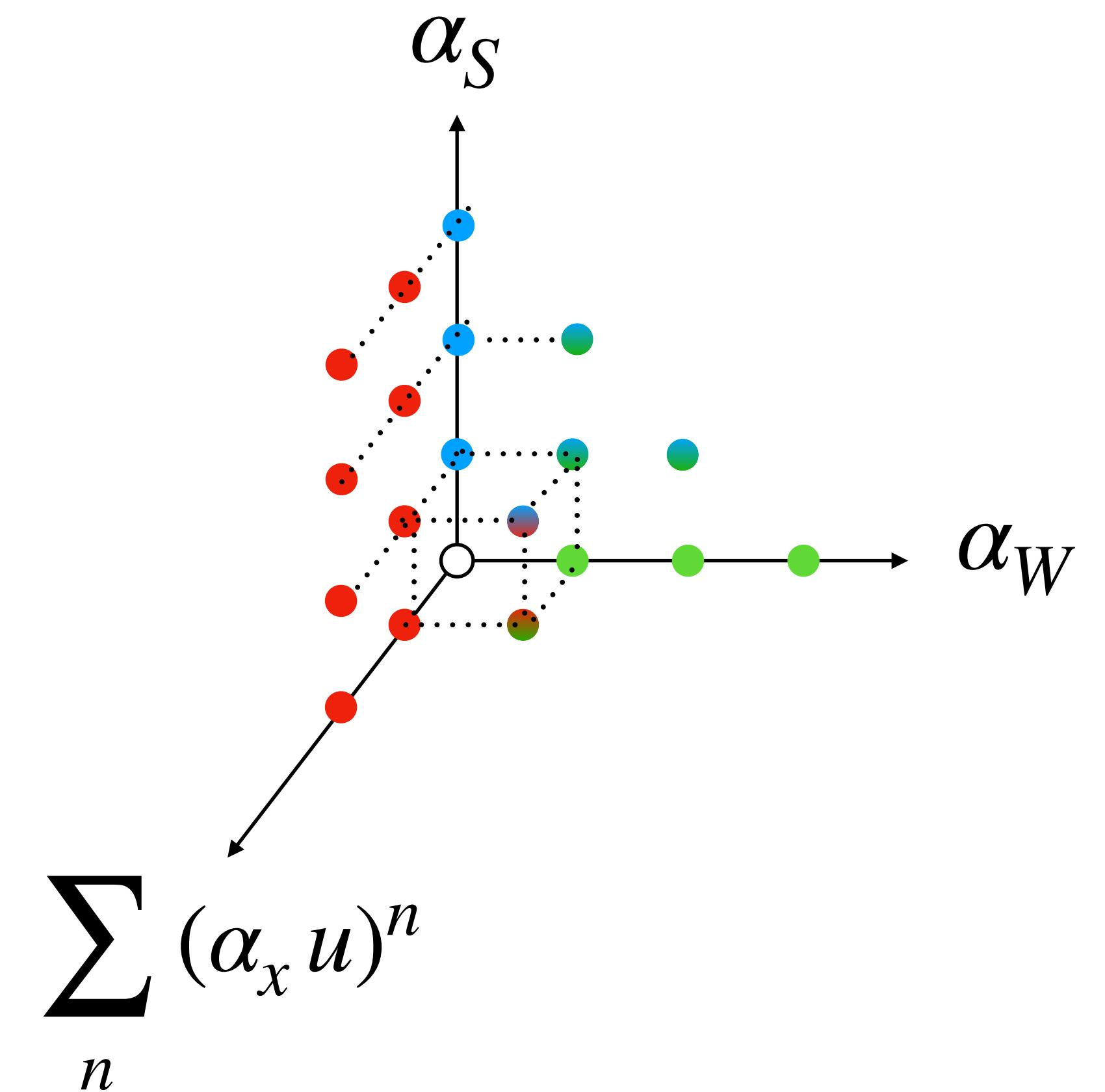
Compute perturbatively

Precision calculations for the LHC

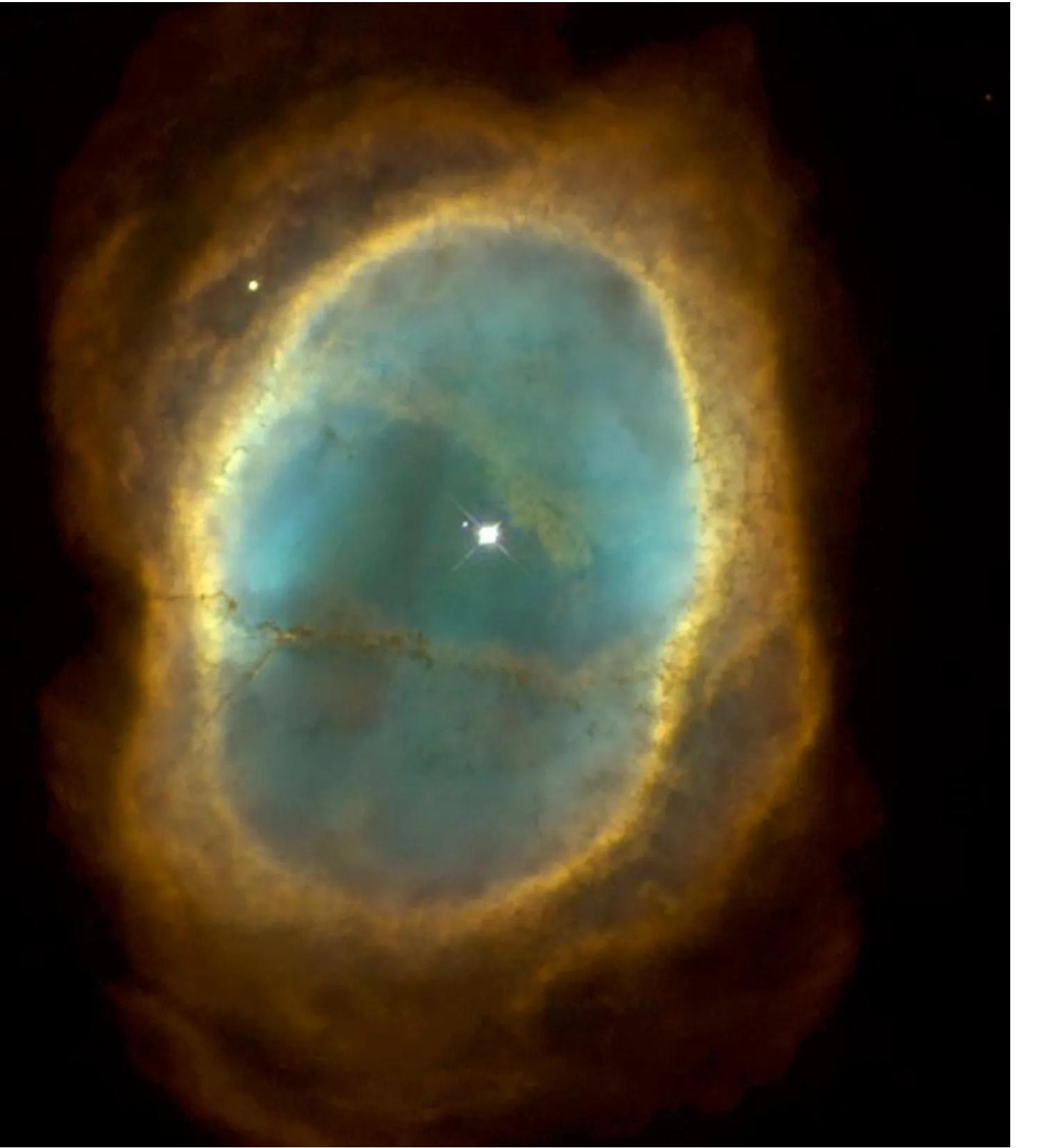
The path

“Rules of thumb at the LHC”:

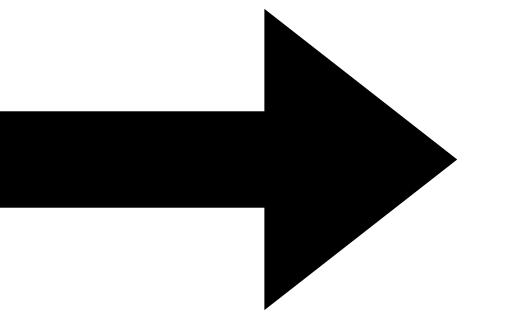
- Predictions must be calculated at least to **NLO QCD** to control the central value at 10-20%.
- **N2LO QCD** provides control at 5% level and on the uncertainties stabilizing the perturbative expansion.
- **N2LO QCD** is expected to be of the **same order** as **NLO EW** $\alpha_S^2 \sim \alpha_W$, yet **EW** corrections grow large and negative at high energies (Sudakov logs).
- **N3LO QCD** is the frontier of precision aiming $\sim 1\%$ of MHO uncertainties.
- **Resummation** Universal, all-order terms that are potentially large for some observables (logs or 1PI loops for propagators) need to be resummed. They might refer to global or non-global observables. Resummation leads to improvements in precision and accuracy.



The precision goal



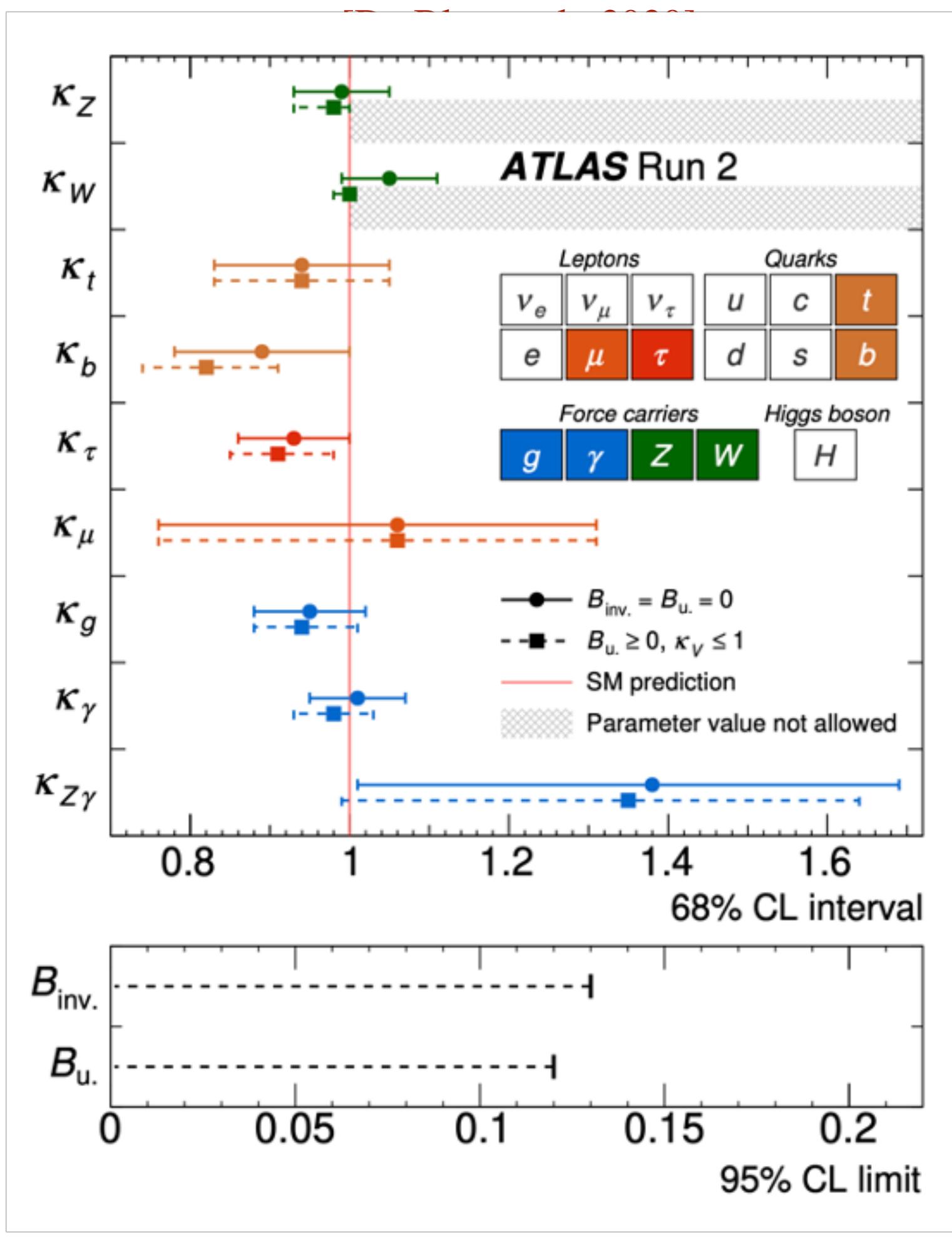
Edwin 1990



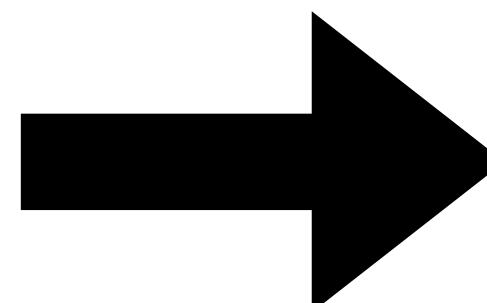
Edwin 2022

HL-LHC projections

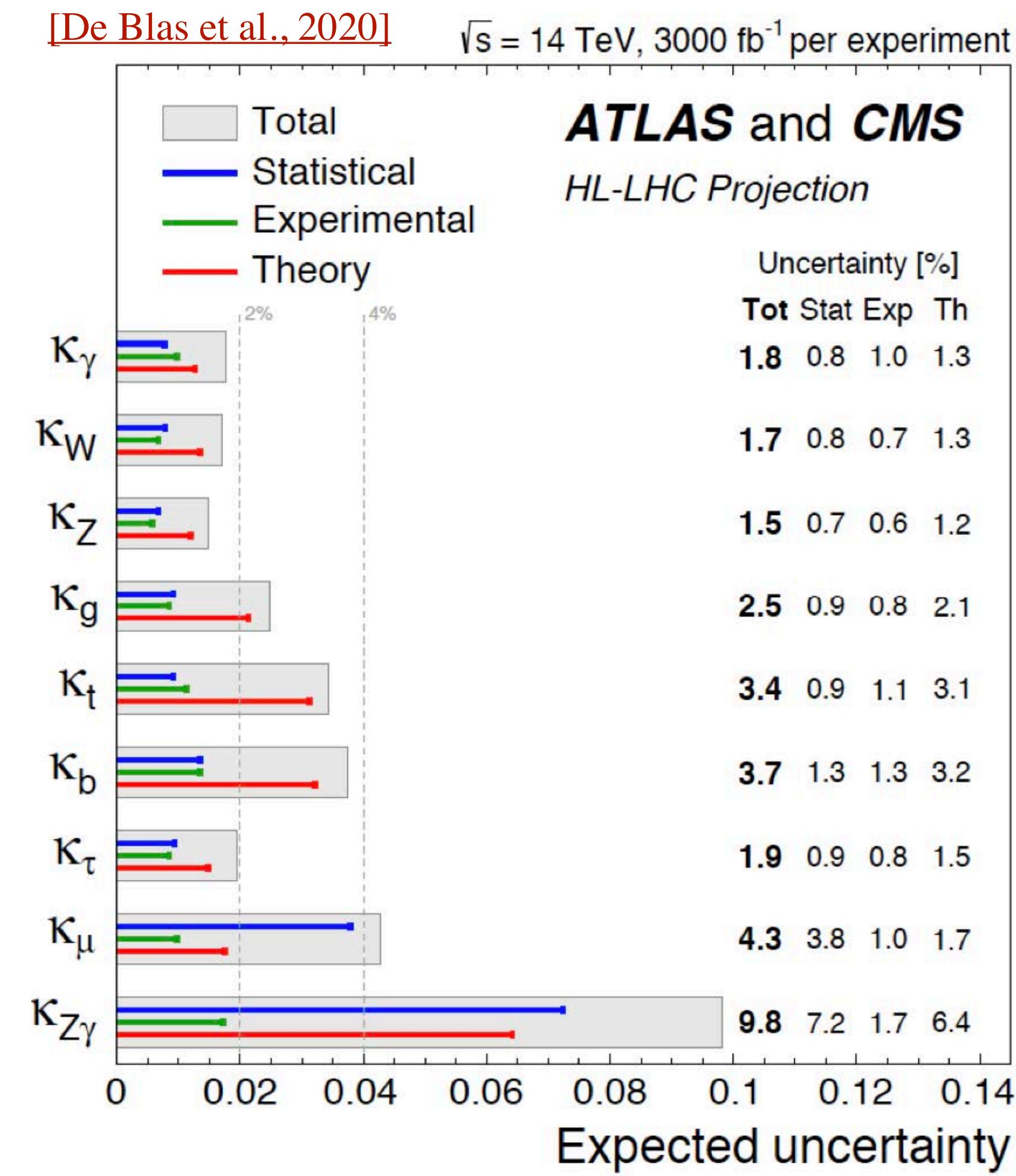
Higgs couplings



10-20%



2-4%

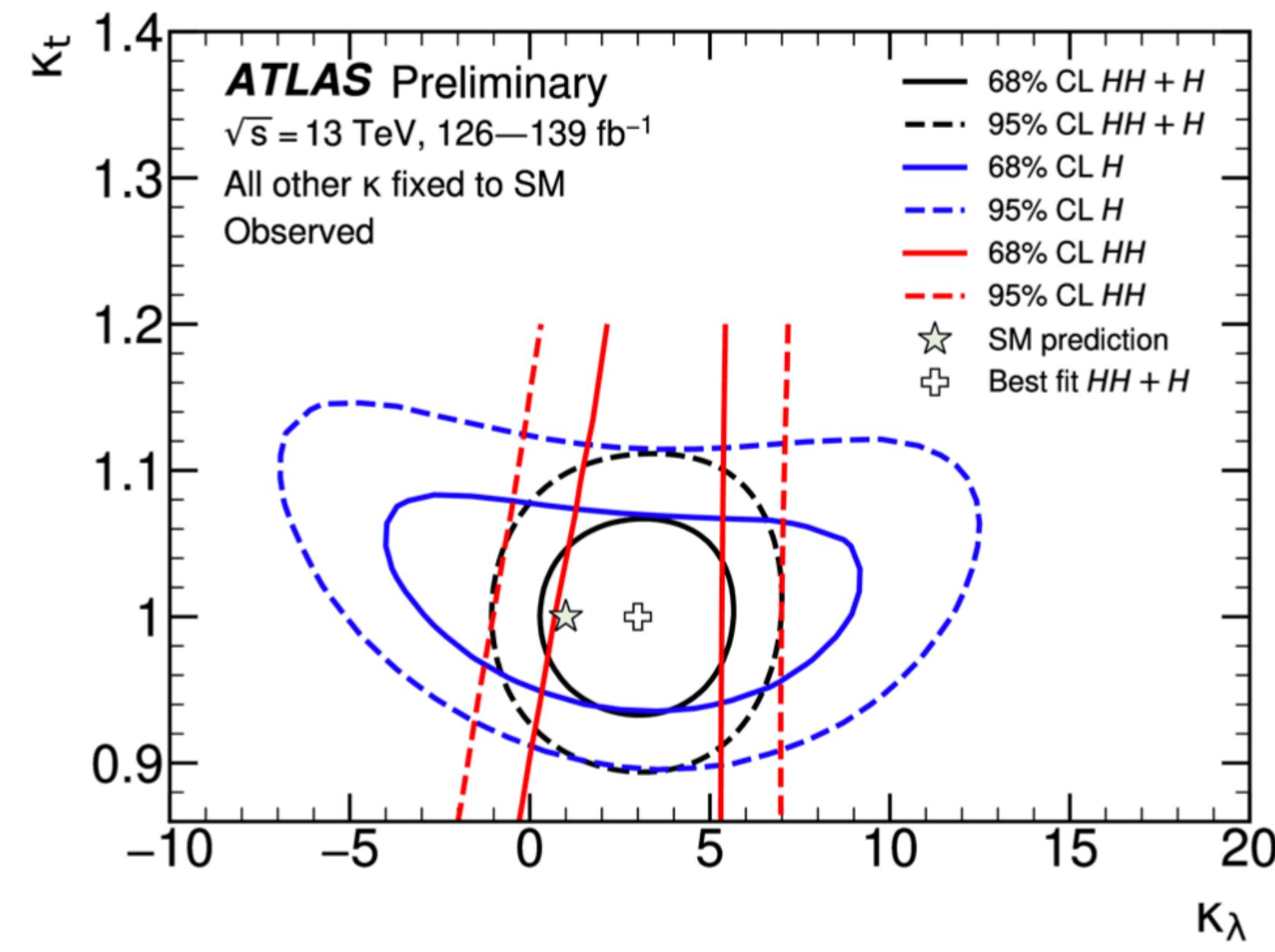


HL-LHC projections

Higgs couplings

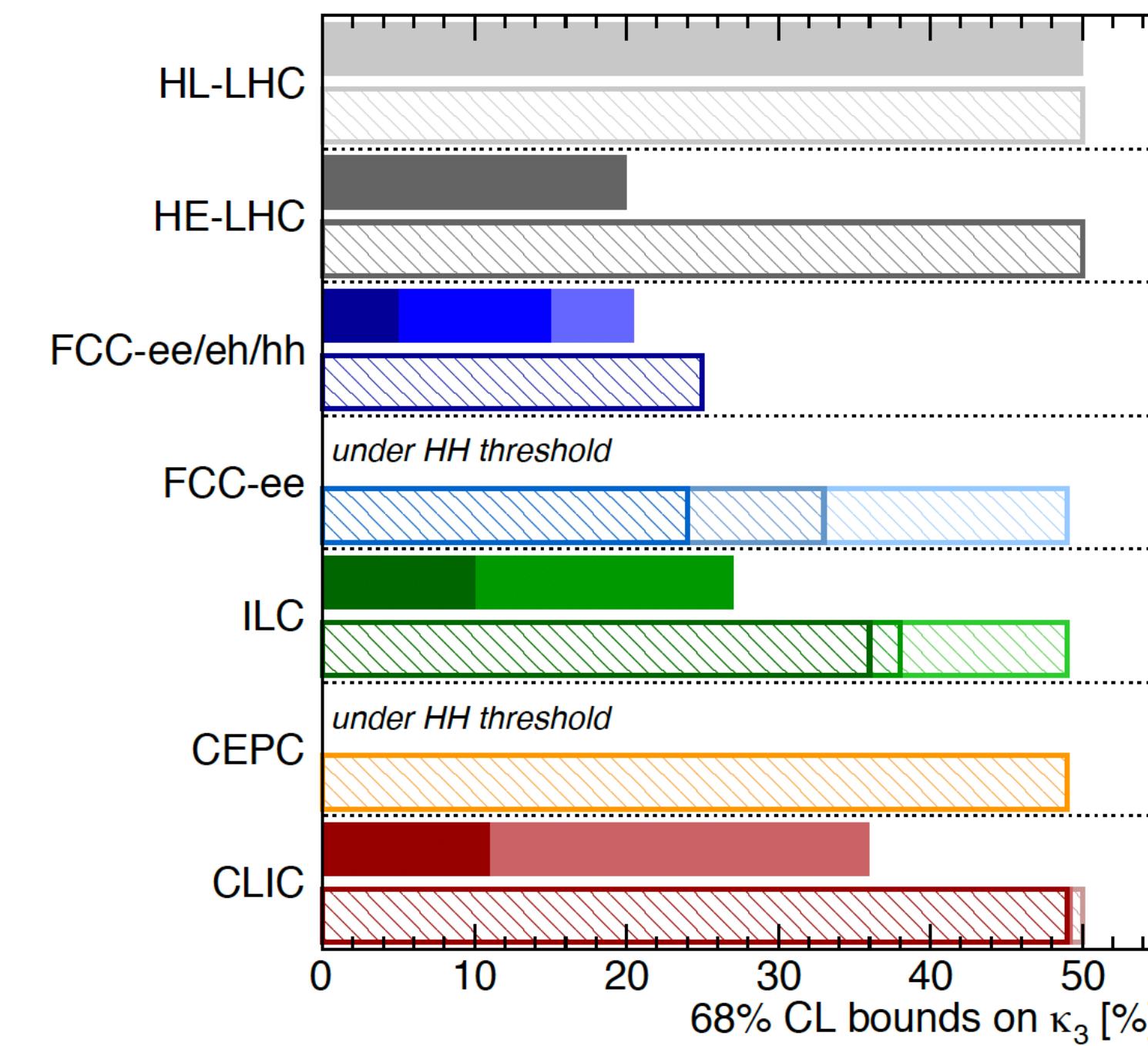
Now

[\[ATLAS, 2022\]](#)



Future

[\[De Blas et al., 2020\]](#)



Higgs@FC WG September 2019

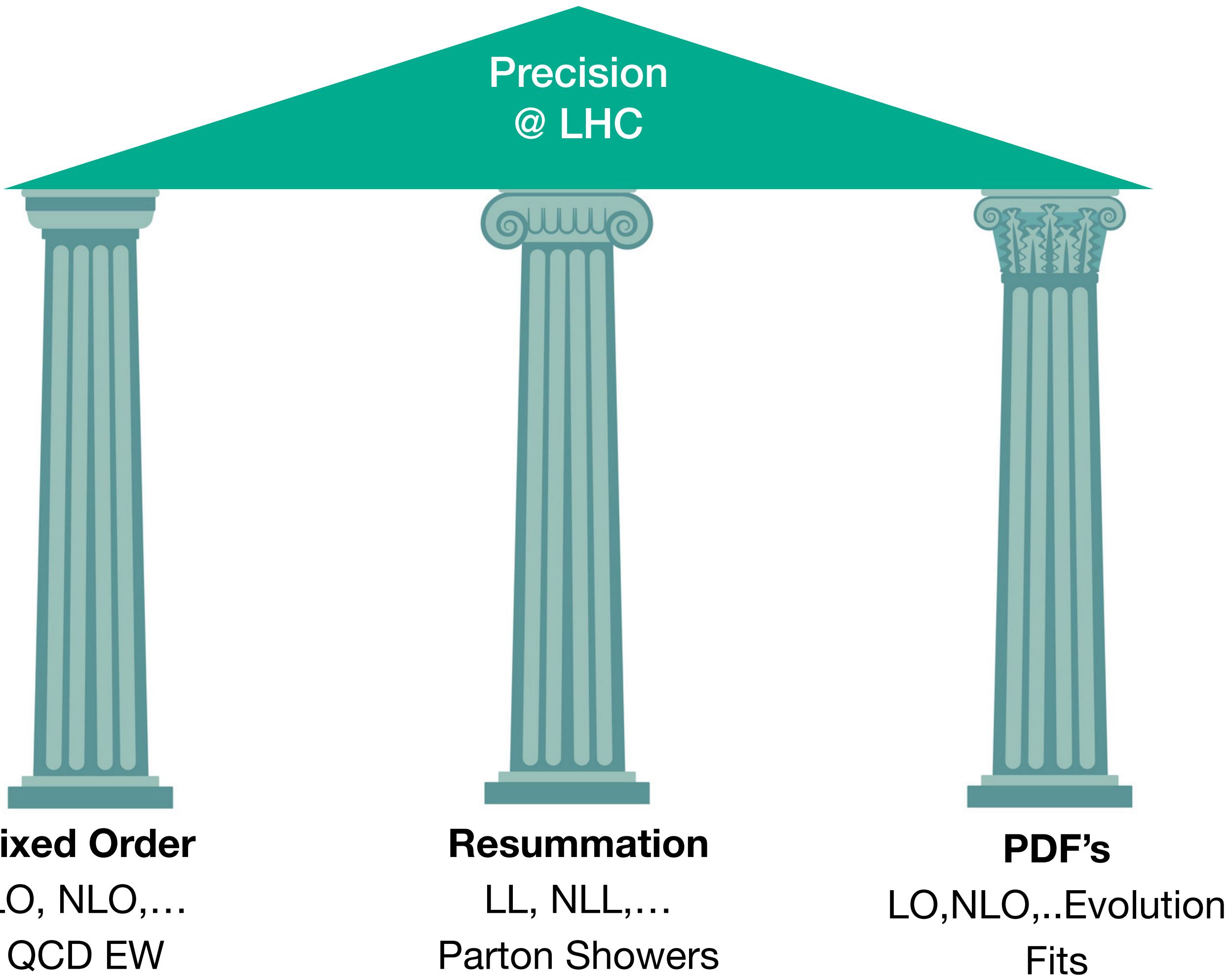
di-Higgs	single-Higgs
HL-LHC	HL-LHC 50% (47%)
HE-LHC	HE-LHC 50% (40%)
FCC-ee/eh/hh	FCC-ee/eh/hh 25% (18%)
LE-FCC	LE-FCC n.a.
FCC-eh ₃₅₀₀	FCC-eh ₃₅₀₀ n.a.
FCC-ee ₃₆₅	FCC-ee ₃₆₅ 24% (14%)
FCC-ee ₃₆₅	FCC-ee ₃₆₅ 33% (19%)
FCC-ee ₂₄₀	FCC-ee ₂₄₀ 49% (19%)
ILC ₁₀₀₀	ILC ₁₀₀₀ 36% (25%)
ILC ₅₀₀	ILC ₅₀₀ 38% (27%)
ILC ₂₅₀	ILC ₂₅₀ 49% (29%)
CEPC	CEPC 49% (17%)
CLIC ₃₀₀₀	CLIC ₃₀₀₀ 49% (35%)
CLIC ₁₅₀₀	CLIC ₁₅₀₀ 49% (41%)
CLIC ₃₈₀	CLIC ₃₈₀ 50% (46%)

All future colliders combined with HL-LHC

Currently limits on k_λ from H and HH are comparable and will stay so at the HL-LHC.
 Borderline sensitivity to say something about EW baryogenesis...

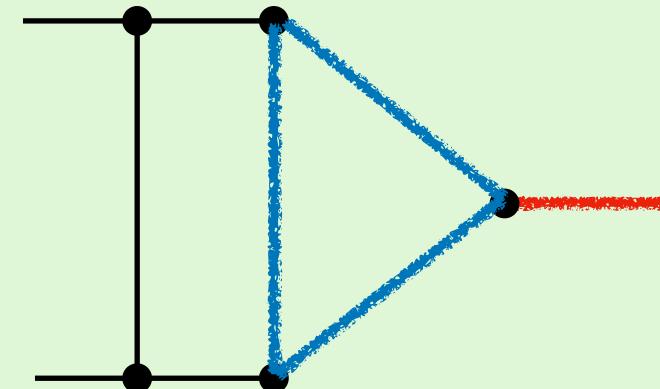
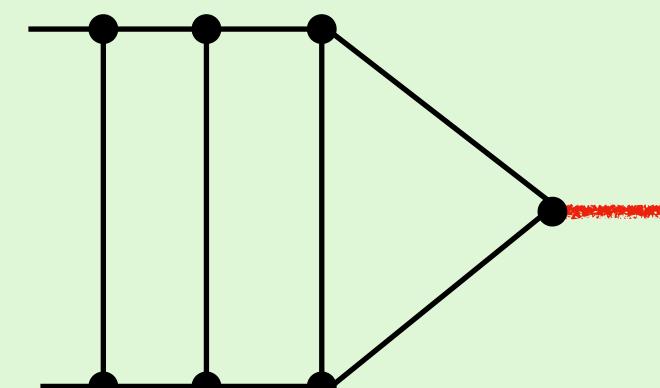
Precision calculations for the LHC

Status

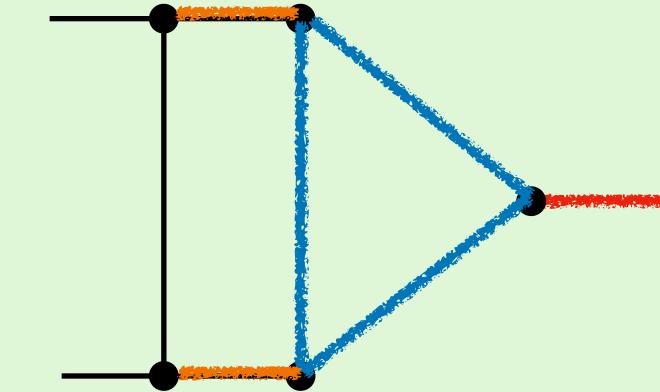
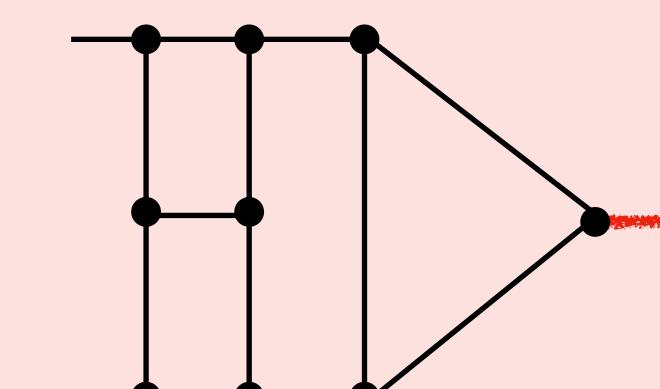
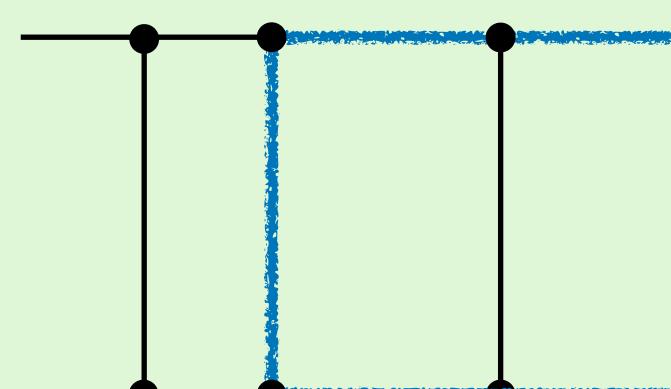
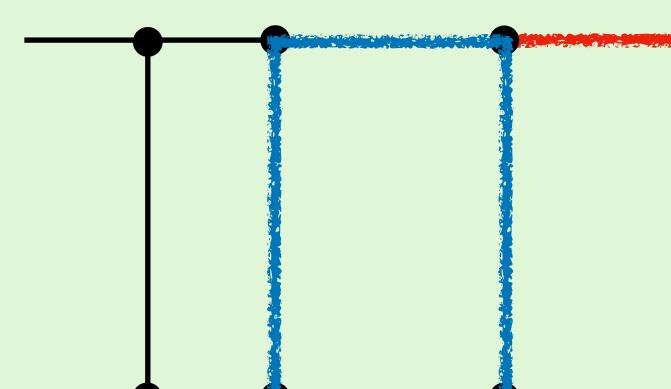
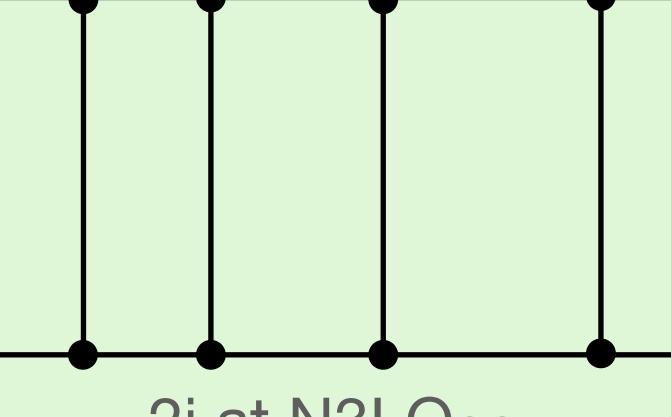
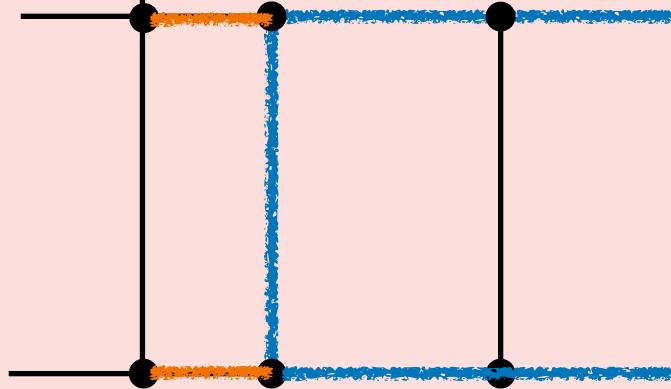
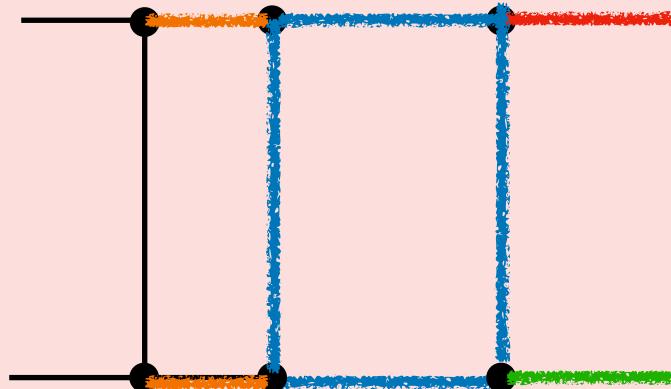
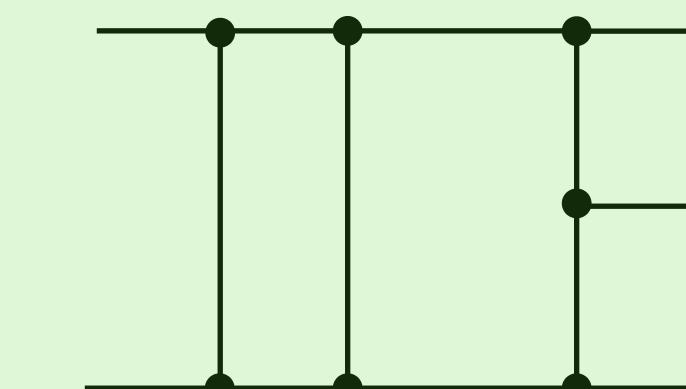
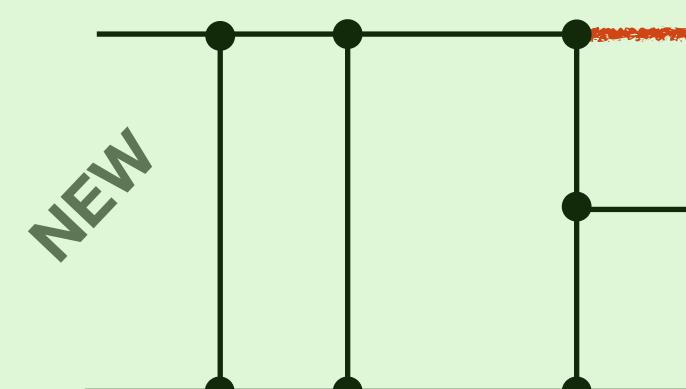
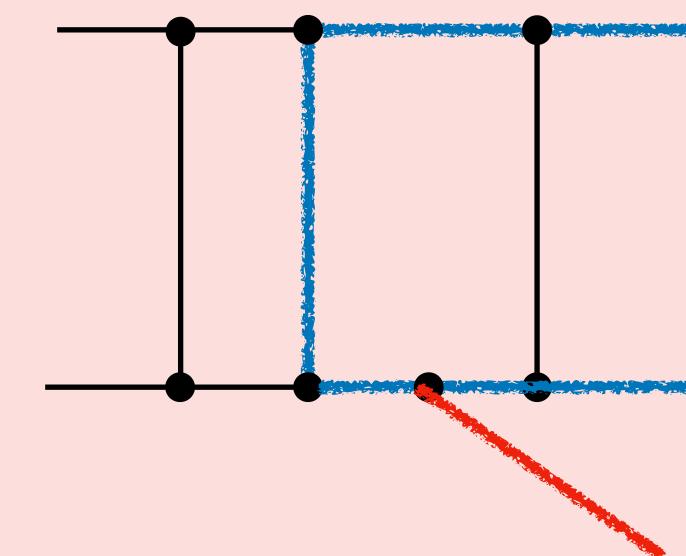


DONE

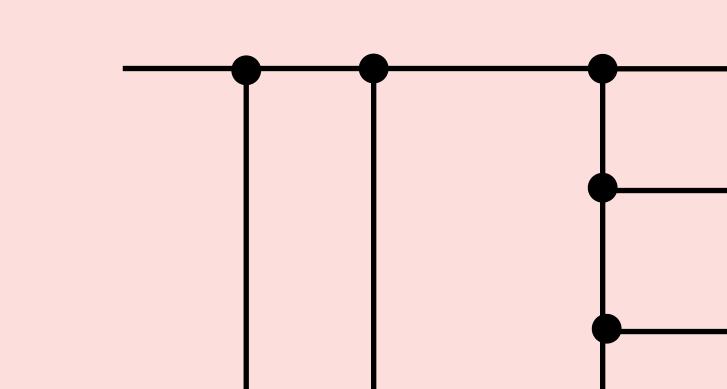
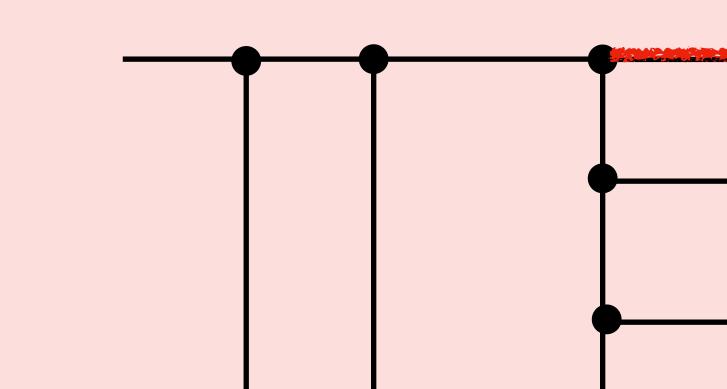
The multi-loop frontier

H,Z,W at $\text{N}2\text{LO}_{\text{QCD}}$ 

H,Z,W at N3LO

H,Z,W at $\text{NLO2}_{\text{EWxQCD}}$ H,Z,W at $\text{N}4\text{LO}_{\text{QCD}}$ tt at $\text{N}2\text{LO}_{\text{QCD}}$ H+j at NLO_{QCD} 2j at $\text{N}3\text{LO}_{\text{QCD}}$ tt at $\text{NLO2}_{\text{EWxQCD}}$ ZH at $\text{N}2\text{LO}_{\text{EW}}$ 3j at $\text{N}2\text{LO}_{\text{QCD}}$ Vbb at $\text{N}2\text{LO}_{\text{QCD}}$ ttH at $\text{N}2\text{LO}_{\text{QCD}}$

- * The more # of loops/legs/scales (colors) the more difficult.
- * Only Z,W,H 2 to 1 production known at N3LO
- * EWxQCD corrections very limited
- * EW N2LO still to be explored
- * Need a subtraction method to turn to IR safe observables

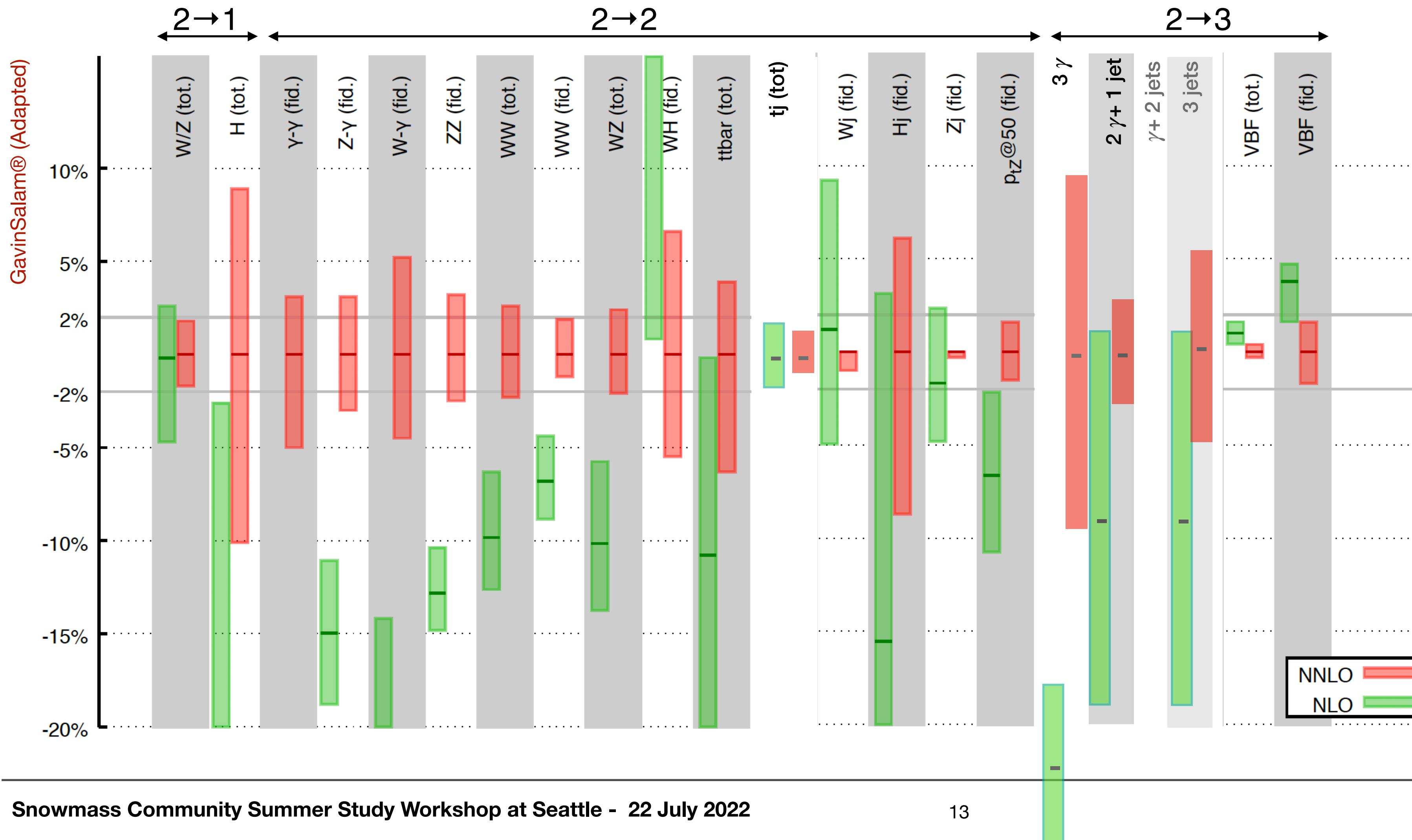
4j at $\text{N}2\text{LO}_{\text{QCD}}$ V+3j at $\text{N}2\text{LO}_{\text{QCD}}$

**As of 22 July 2022.
FAST MOVING FRONTIER**

TO DO

Precision calculations for the LHC

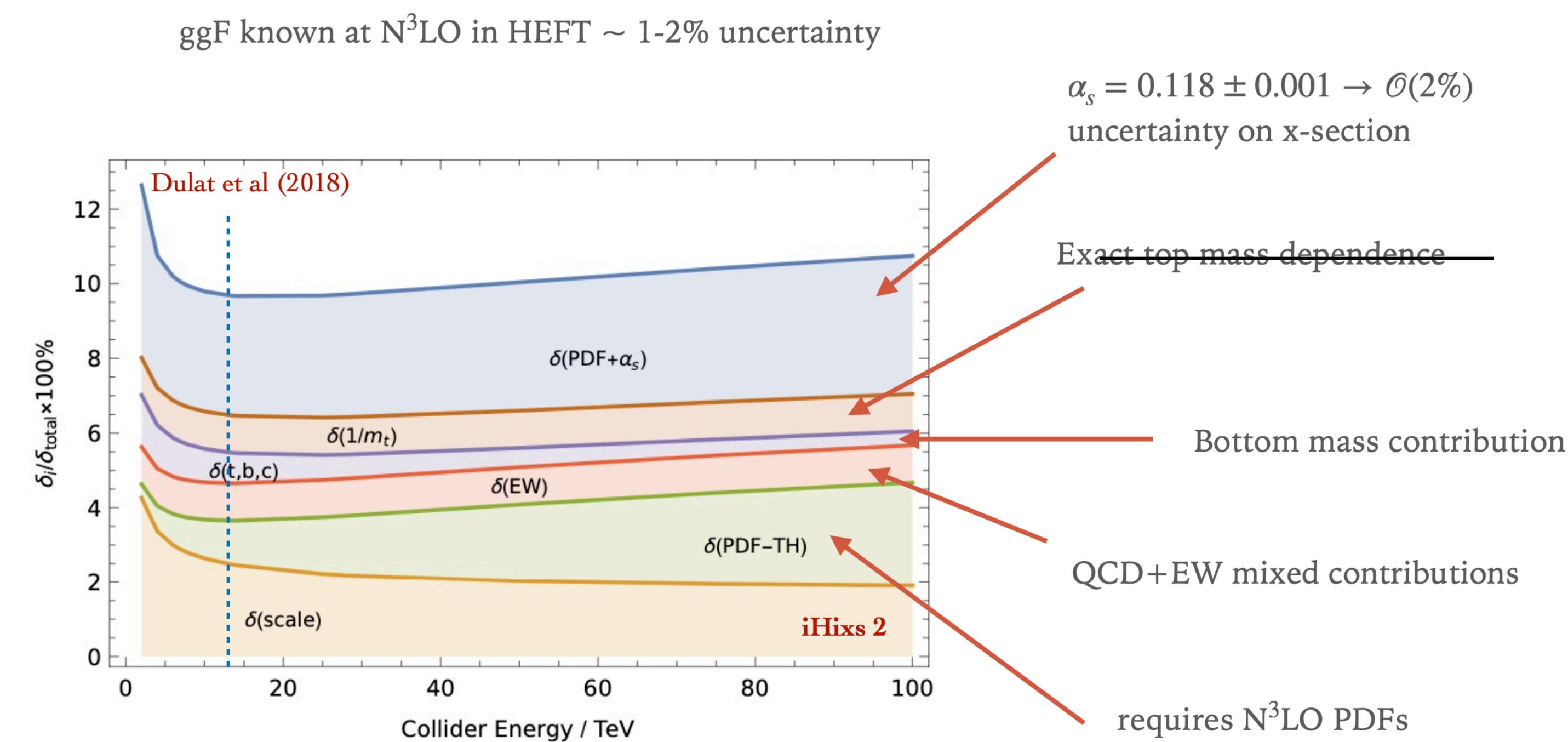
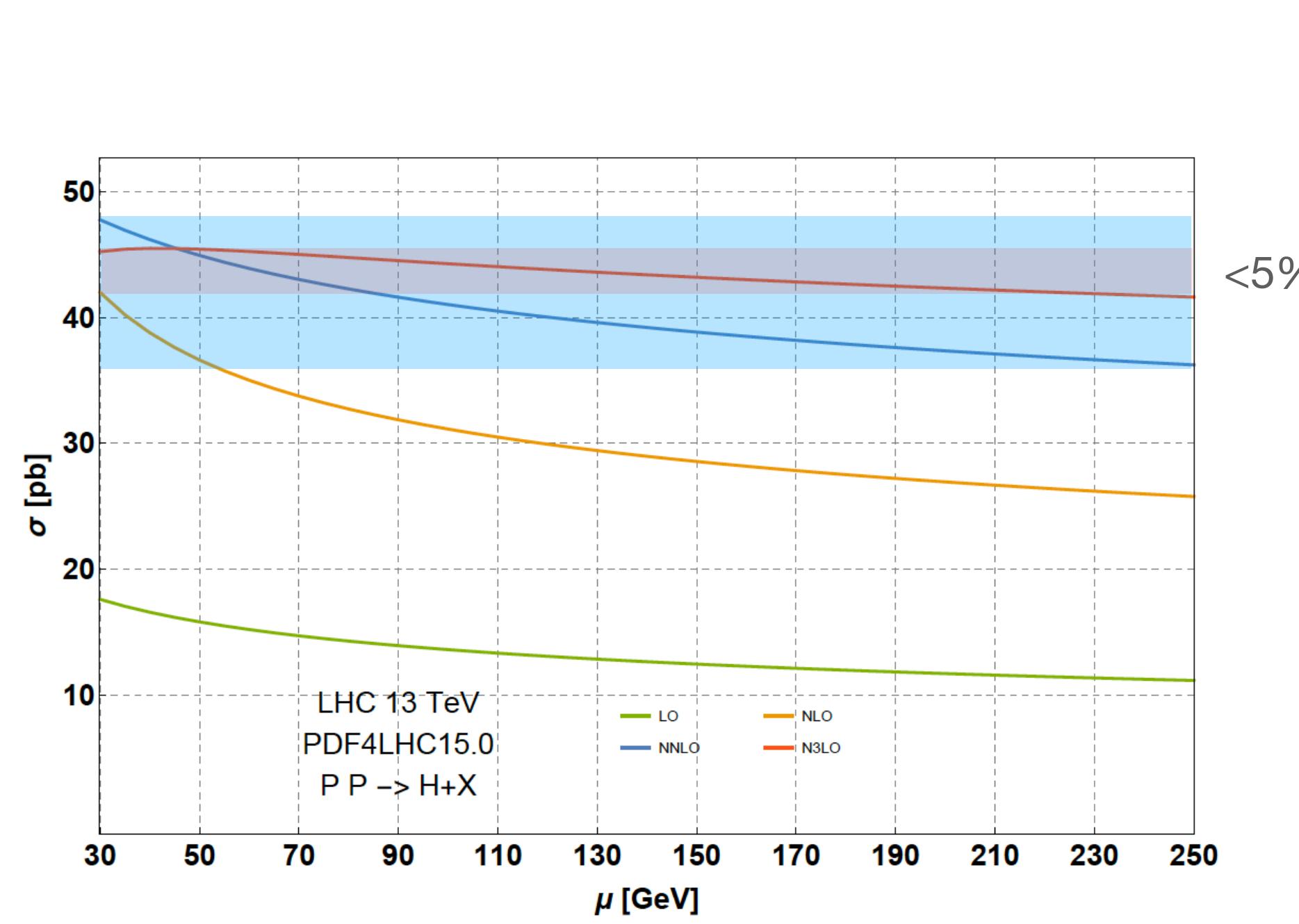
Status: Fixed Order



- NNLO brings us in the few percent arena
- Several NNLO computations move the central value out of the NLO uncertainties
- The 2→3 wall broken

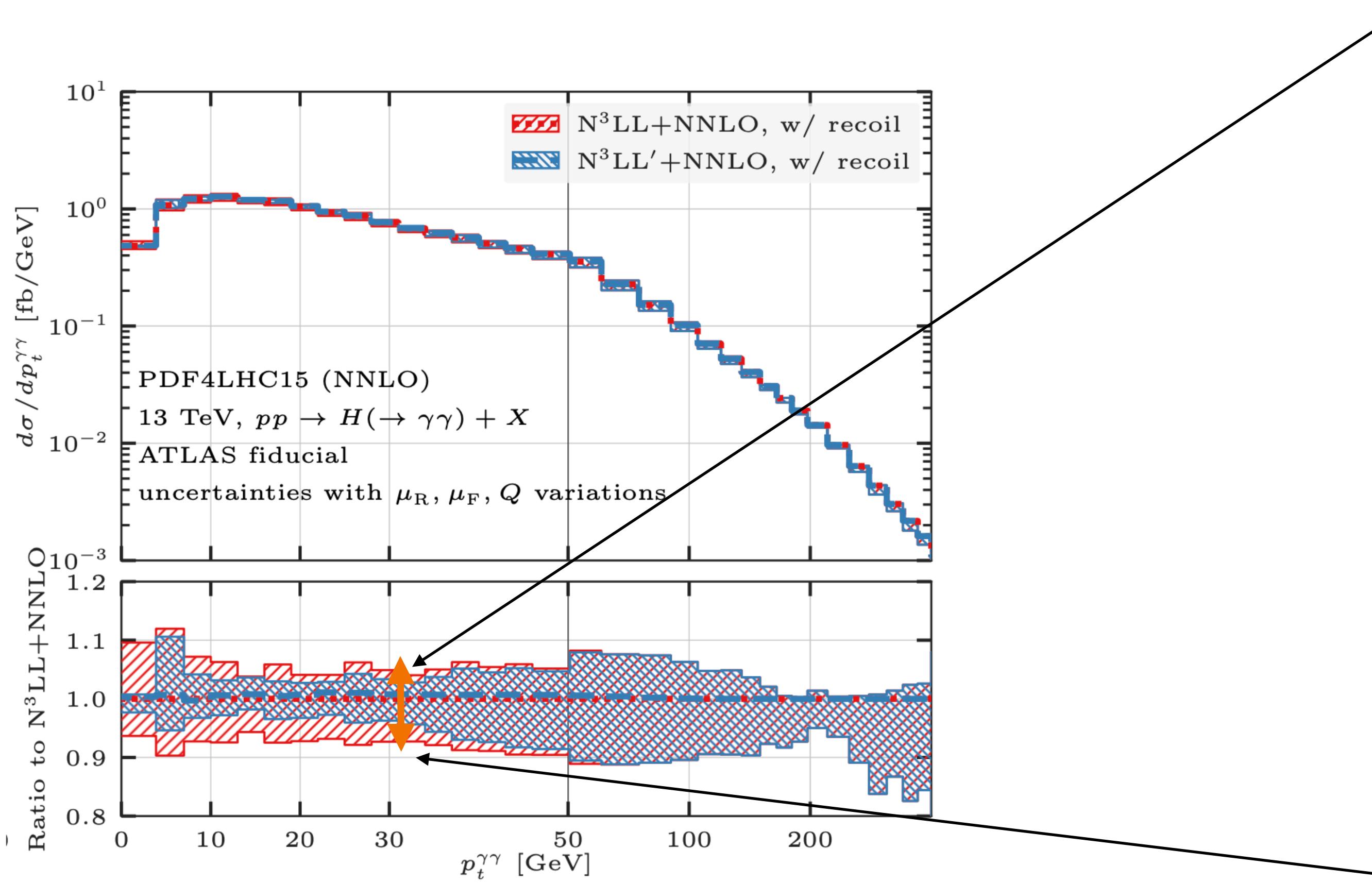
Precision calculations for the LHC

N3LO revolution

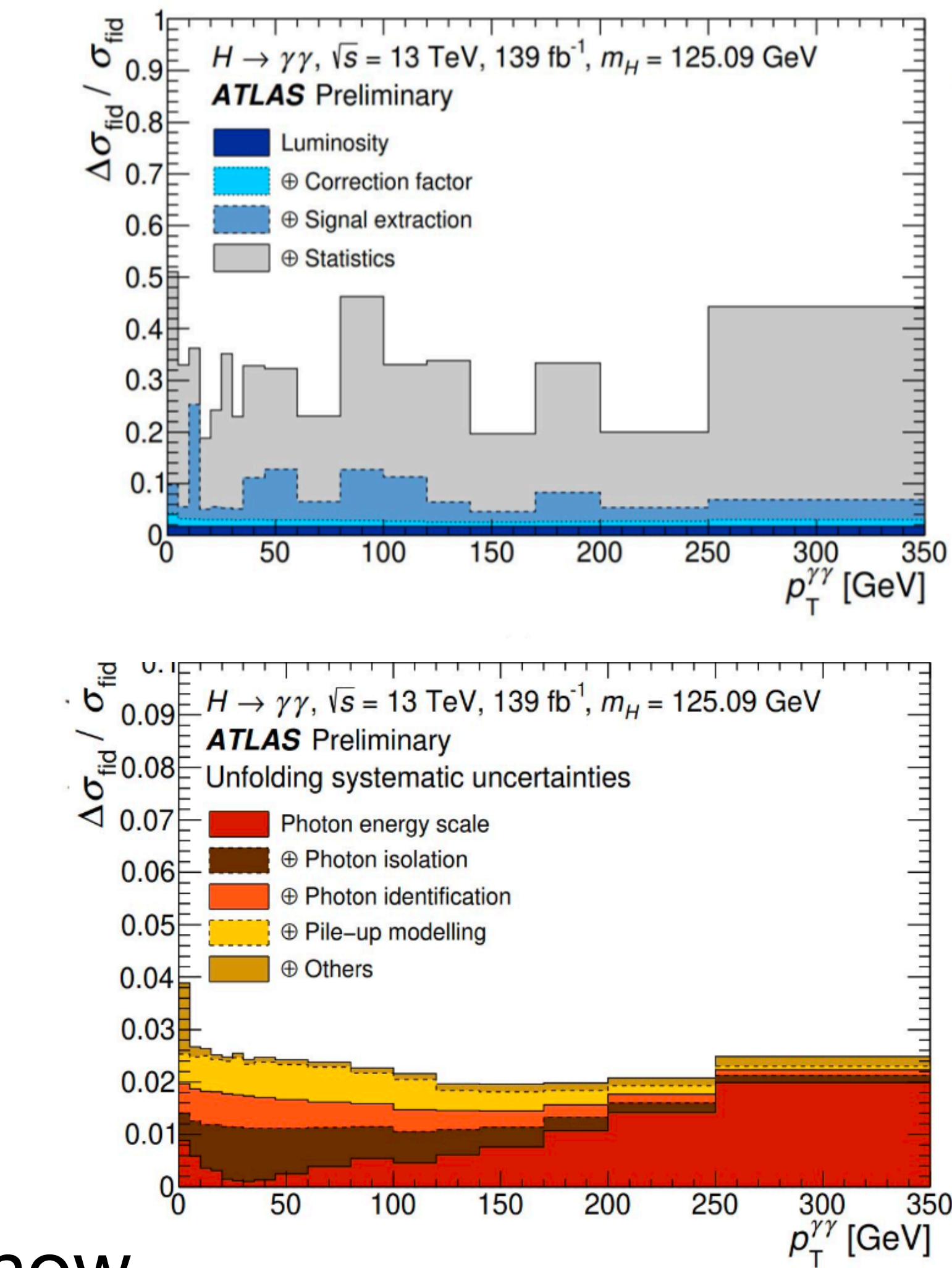


Precision calculations for the LHC

N3LO : good for the moment

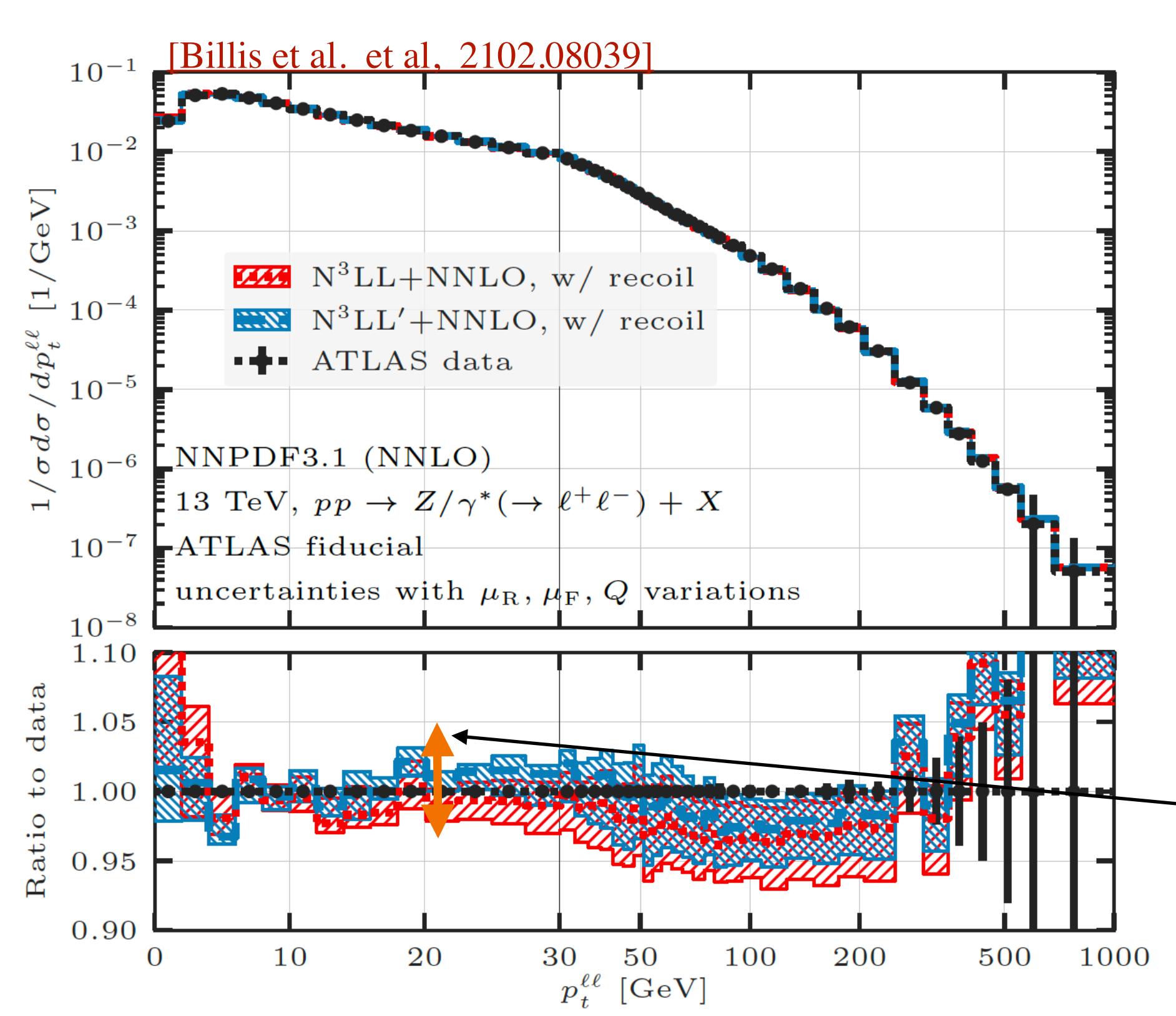


TH and EXP are comparable now....

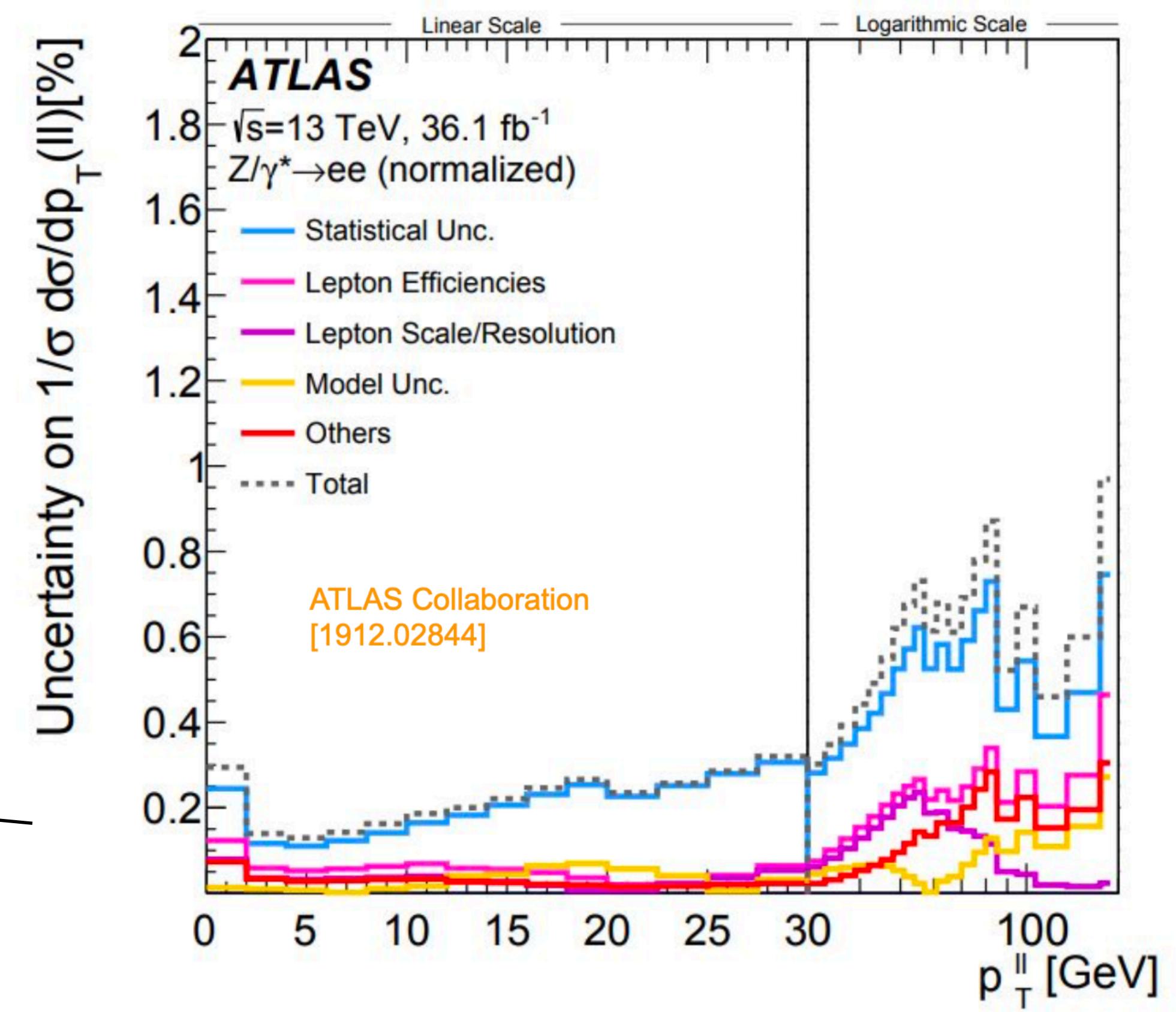


Precision calculations for the LHC

N3LO : already not enough



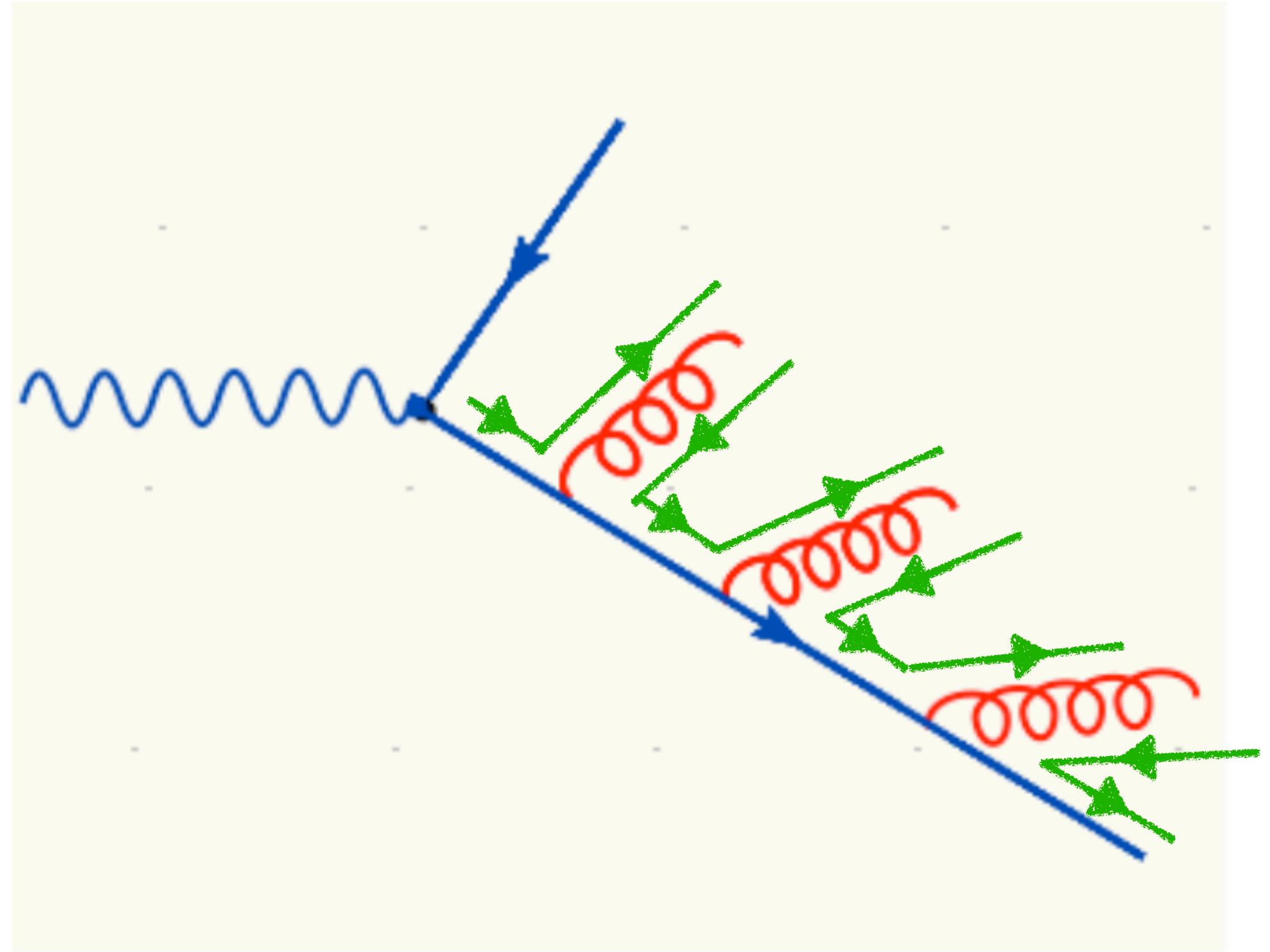
Resummation improves the stability of the cross section predictions even in presence of cut-induced log effects.



EXP (way) better than TH already now!!

Precision calculations for the LHC

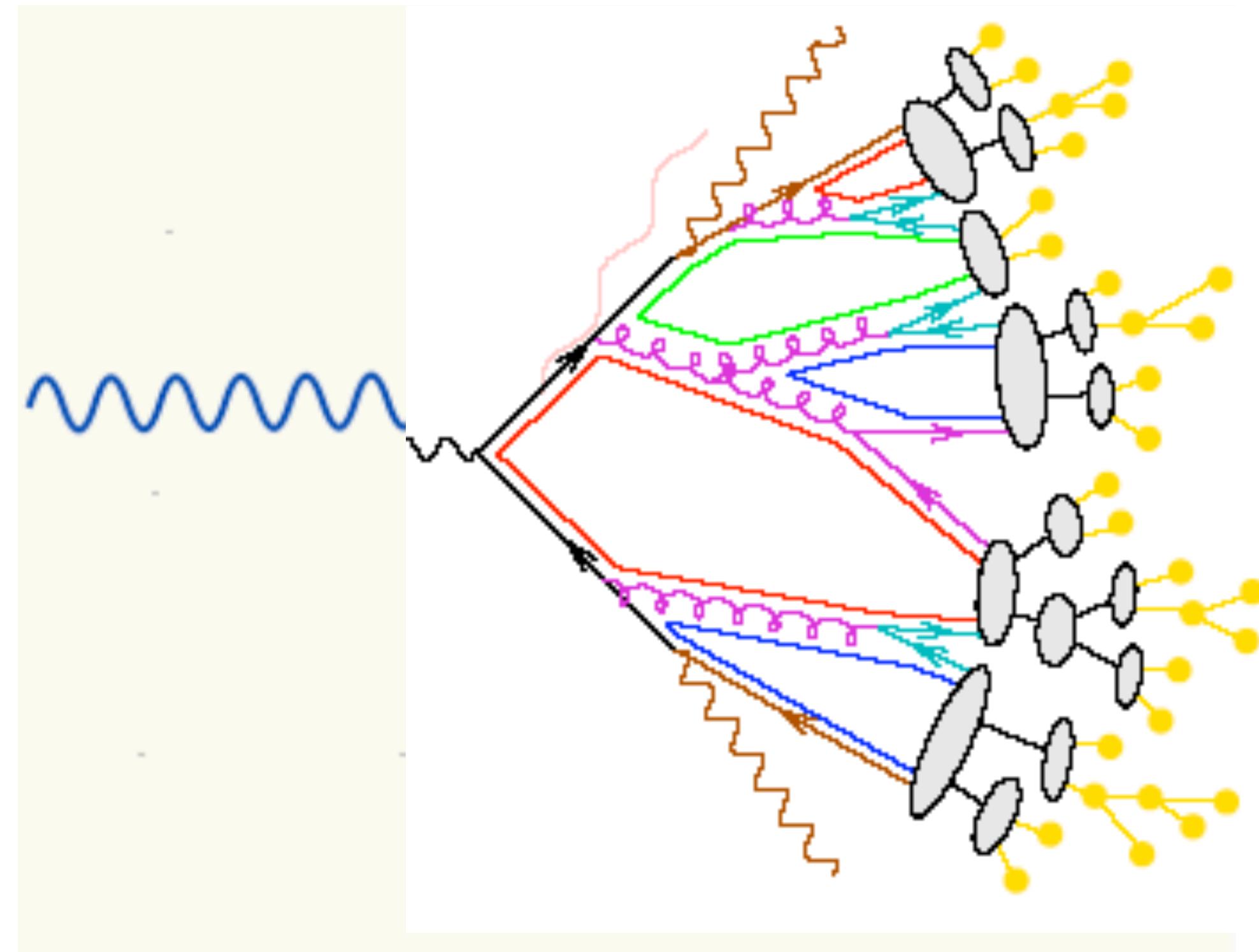
Fully exclusive simulations



In QCD every gluon emission creates a new dipole with a smaller opening. Angular ordering of the soft radiation. Finally leading to color-connected cluster of low virtuality that can fragment into hadrons. DGLAP resummation+angular ordering allows collinear and soft resummation.

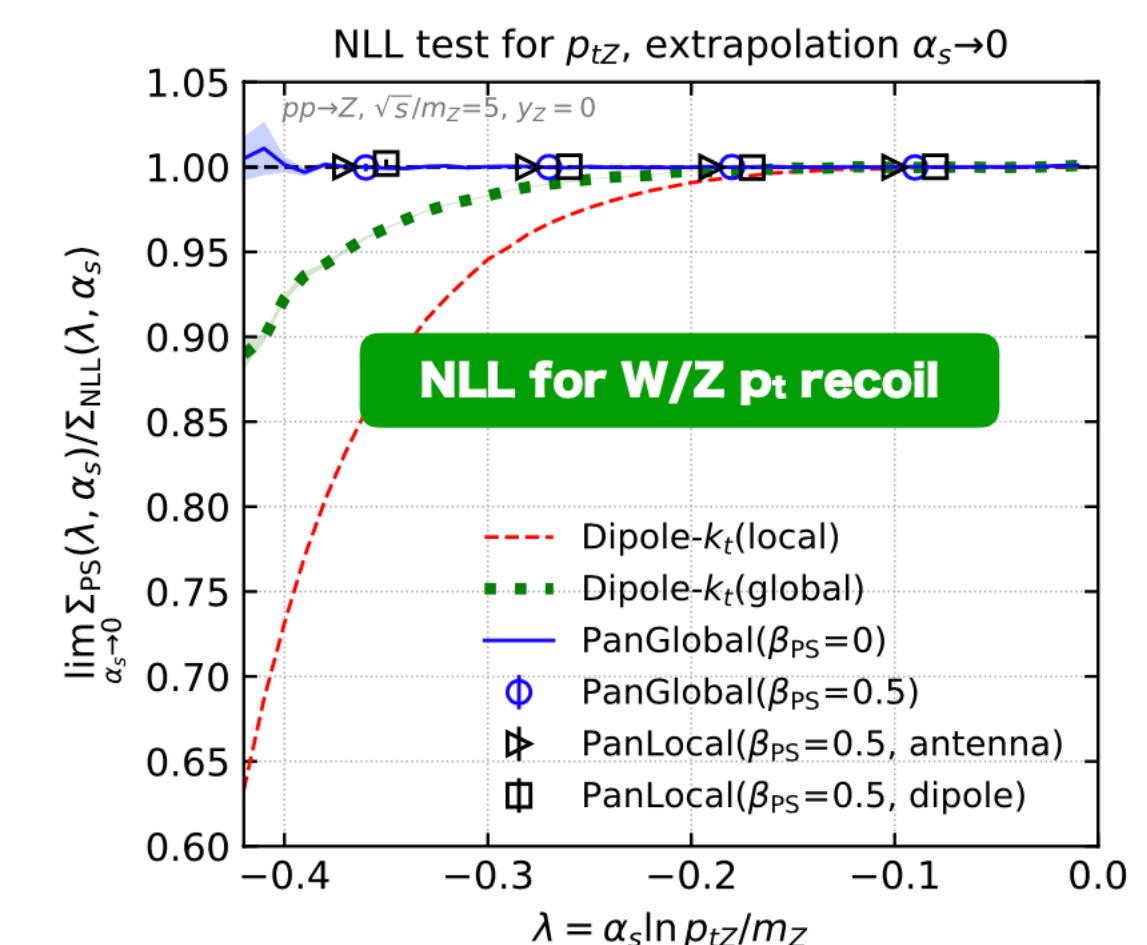
Precision calculations for the LHC

Fully exclusive simulations



In QCD every gluon emission creates a new dipole with a smaller opening. Angular ordering of the soft radiation. Finally leading to color-connected cluster of low virtuality that can fragment into hadrons. DGLAP resummation+angular ordering allows collinear and soft resummation.

All current PS implementations are formally at LL accuracy (with several improvements towards NLL). Moving them to NLL has been proven a formidable challenge. Needs to account subleading effects in the logs and in color.



[\[M. Dasgupta et al. 2002.11114 \]](#) [\[K. Hamilton et al. 2011.10054 \]](#)

Systematic explorations are on-going and very promising.

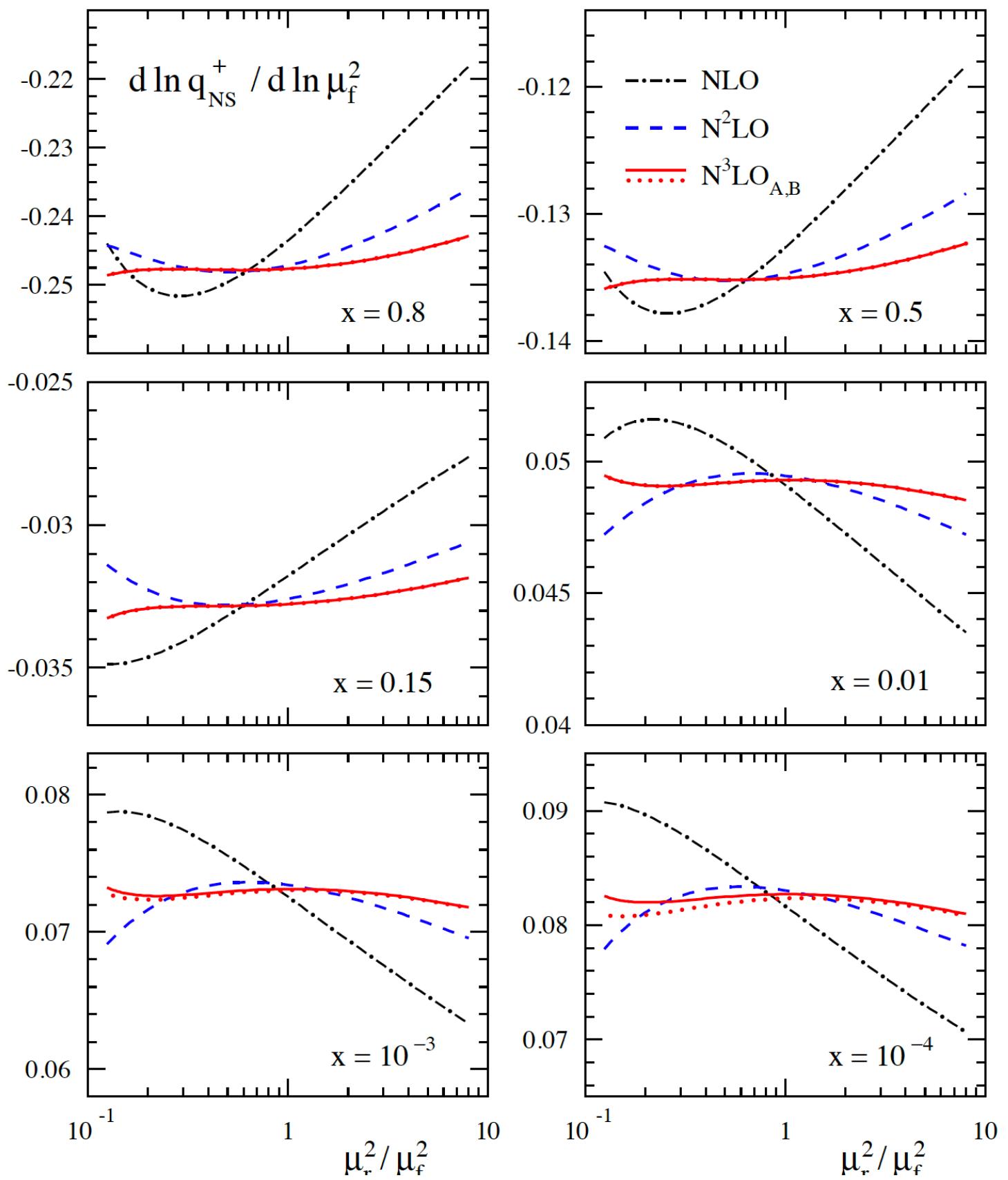
[\[Nagy and Soper 2011.04773, 2011.04777\]](#)

[\[Forshaw et al. 2003.06400\]](#)

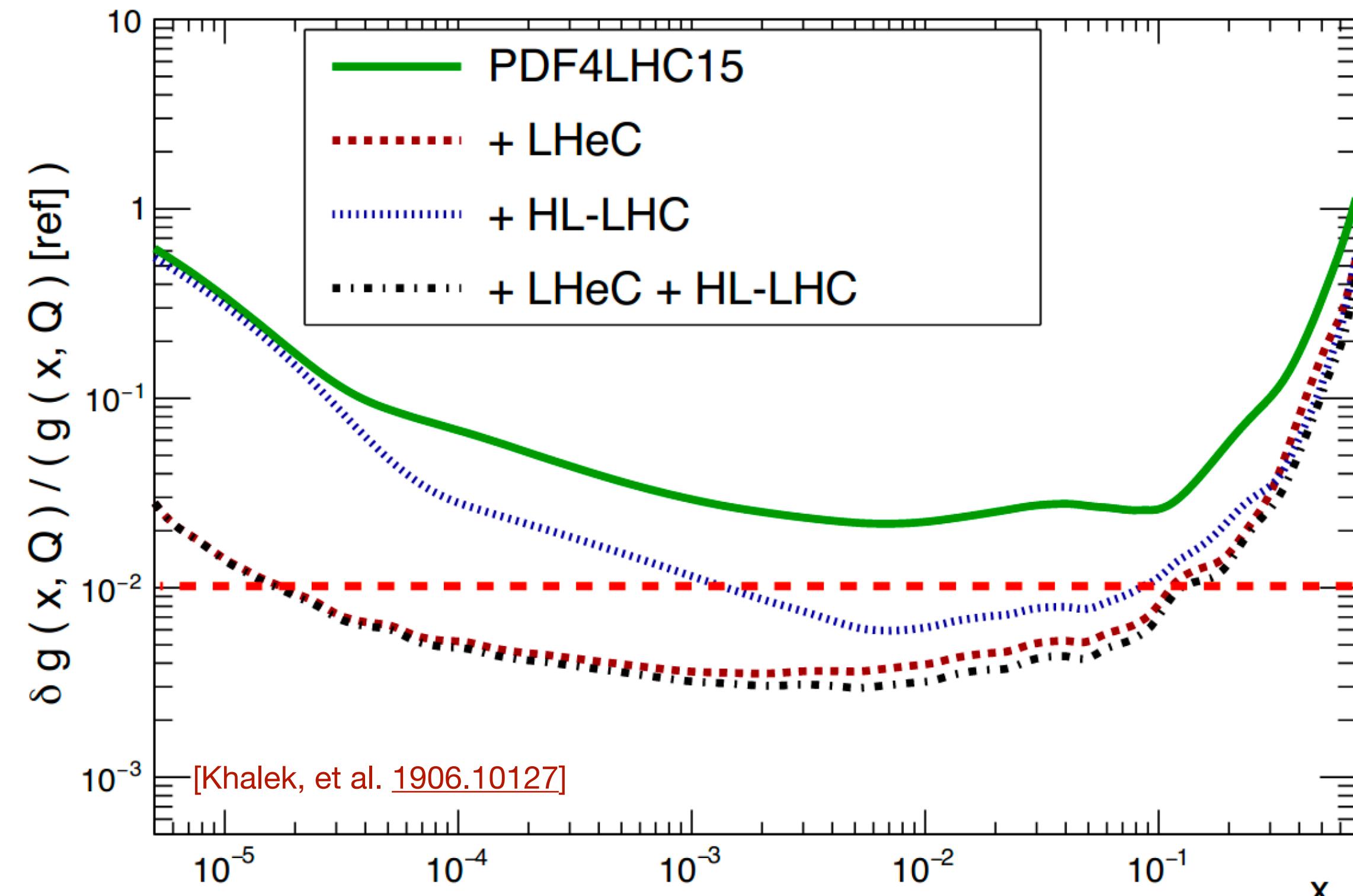
- and much more, e.g.
- conceptually, practically simple soft spin correlations
(Hamilton, Karlberg, GPS, Scyboz, Verheyen [2111.01161](#))
 - calculations for steps beyond NLL
 - collinear splitting: Dasgupta, El Menoufi@ICHEP, [2109.07496](#);
 - subleading non-global logs: Banfi, Dreyer, Monni, [2111.02413](#); NNDL
 - multiplicity at NNDL: Medves@ICHEP, Soto-Ontoso, Soyez, [2111.02413](#)
 - phenomenological use (a year or two away)

Precision calculations for the LHC

Status: PDF's

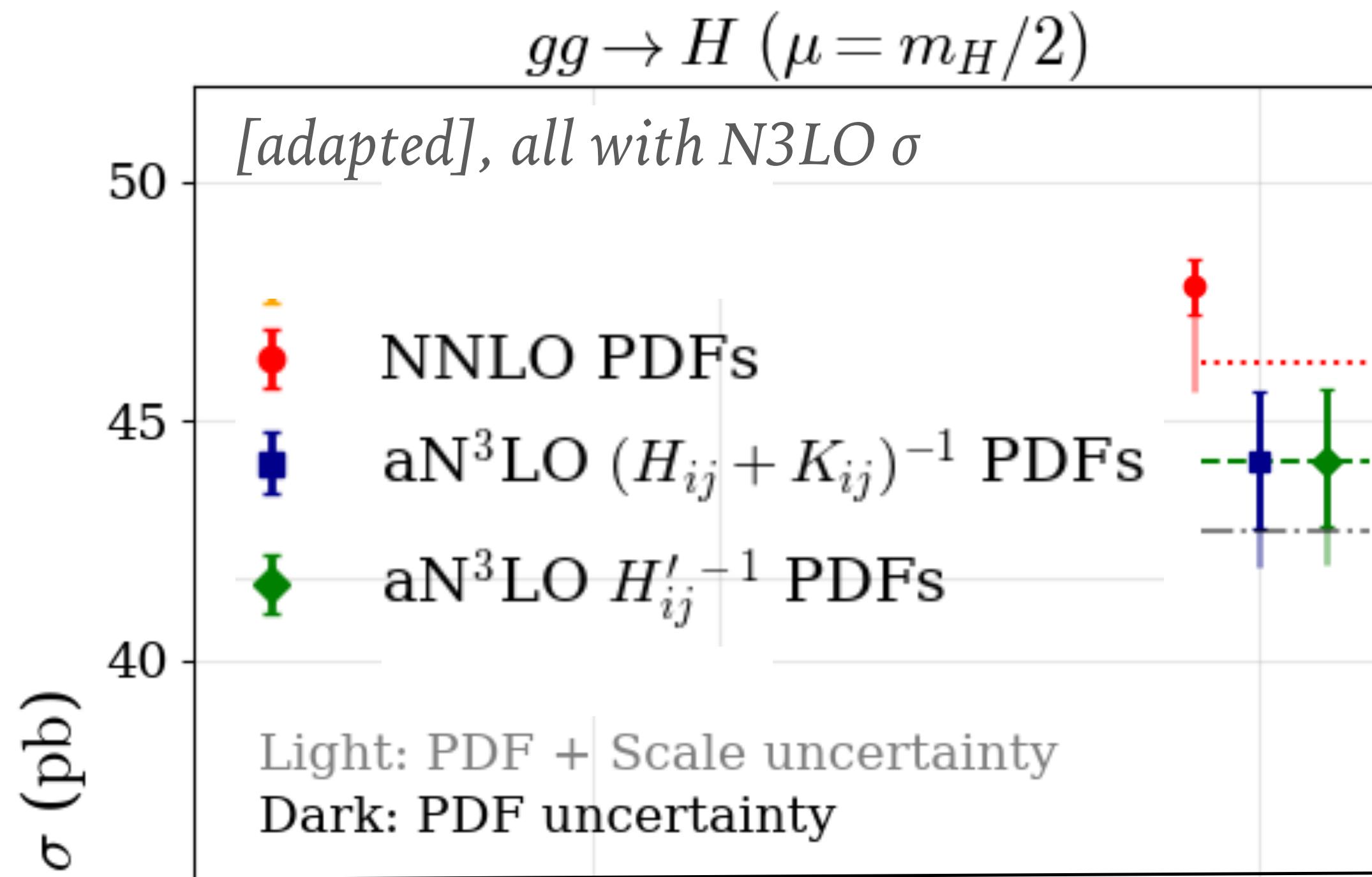


- Complete N3LO PDF's evolution not available yet. Non-singlet evolution available at 4 loops already.
- Error budget with many sources. MHO uncertainties yet to be included in the final assessment.
- Reaching 1% will be very challenging.
- Room for a breakthrough from lattice?



Precision calculations for the LHC

Status: PDF @ N3LO

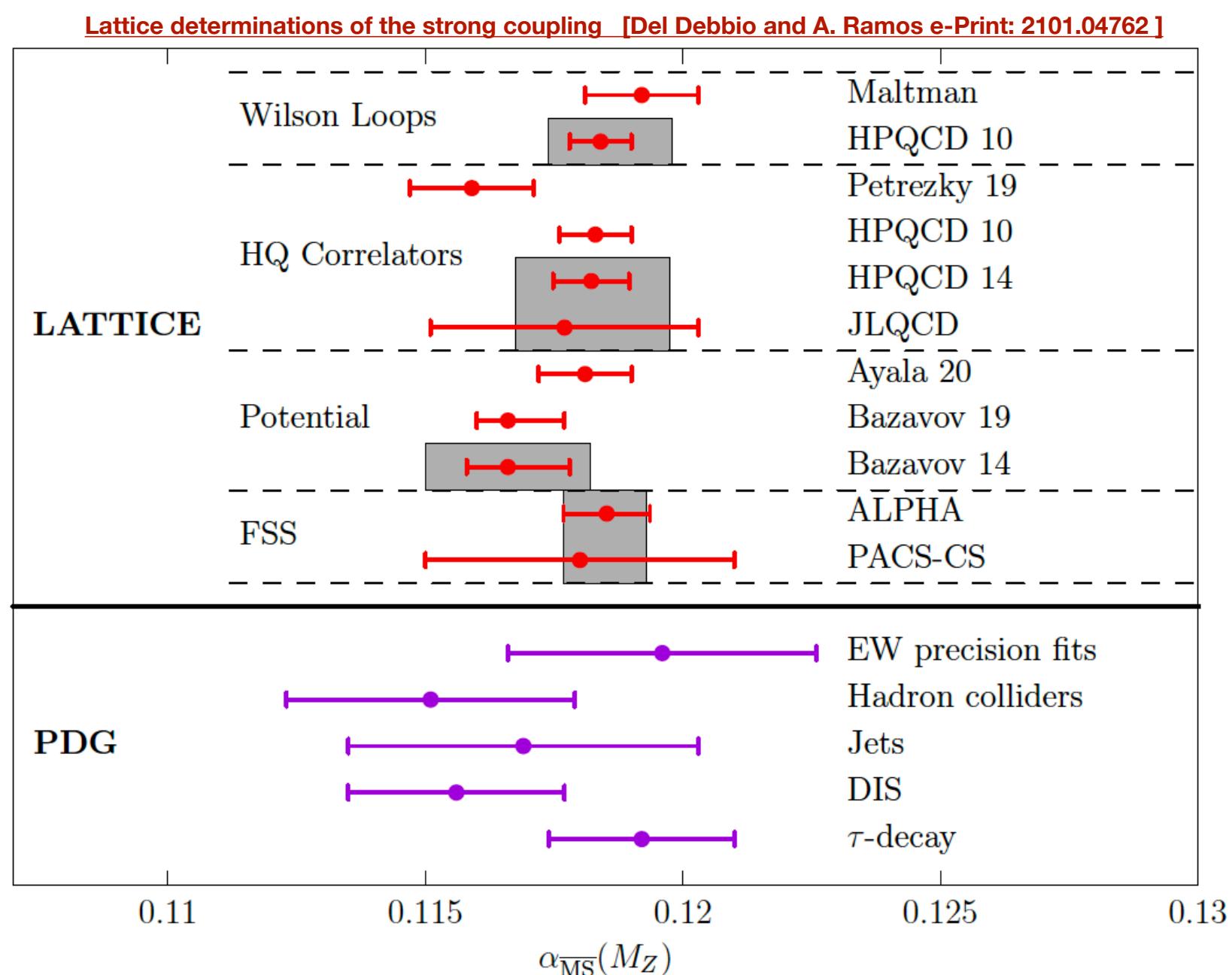


[J. McGowan et al 2207.04738]

- Includes approximations and data-driven fits to N3LO parts that are current unwon
- Sizeable decrease in Higgs cross sections (~8%)
- Leads to a larger uncertainties in the PDF

σ order	PDF order	σ (pb) + $\Delta\sigma_+$ - $\Delta\sigma_-$ (%)
		PDF uncertainties
N ³ LO	aN ³ LO (no theory unc.)	44.164 + 3.03% - 3.13%
	aN ³ LO $(H_{ij} + K_{ij})$	44.164 + 3.34% - 3.15%
	aN ³ LO (H'_{ij})	44.164 + 3.43% - 3.07%
	NNLO	47.817 + 1.17% - 1.22%

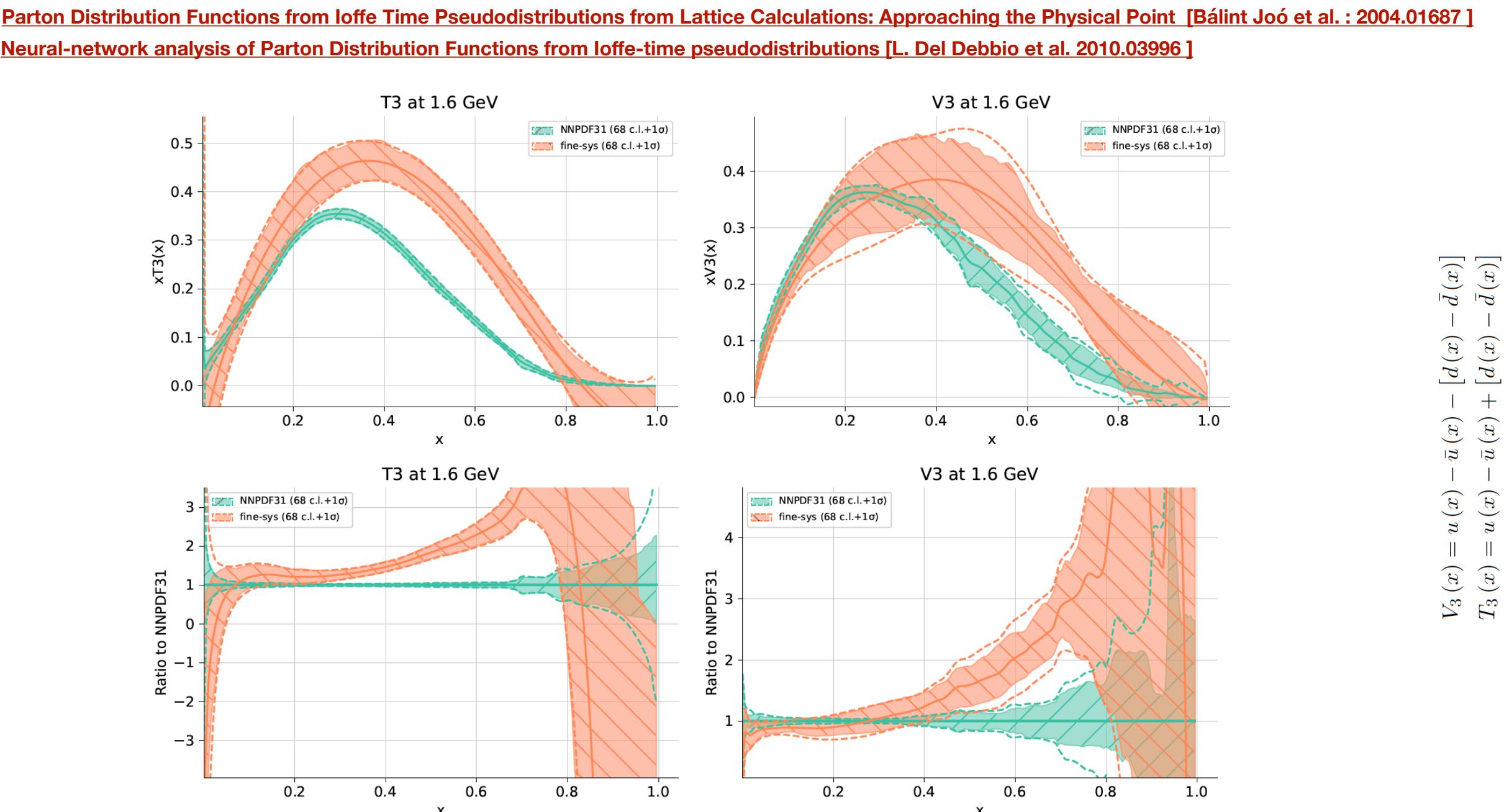
The lattice frontier α_s and PDF's



$$P(Q) = \sum_{k=0}^n c_k(s) \alpha_{\overline{\text{MS}}}^k(\mu) + \mathcal{O}(\alpha_{\overline{\text{MS}}}^{n+1}(\mu)) + \mathcal{O}\left(\frac{\Lambda^p}{Q^p}\right), \quad (s = \mu/Q).$$

MHO		PC
-----	--	----

Using Lattice QCD, one can combine input from well-measured QCD quantities -- like for example the proton mass, or a meson decay constant -- with the perturbative expansion of a short distance observable that does not need to be directly observable (like the quark anti-quark force). The advantage of this approach is that the experimental input comes from the hadron spectrum with a negligible uncertainty.

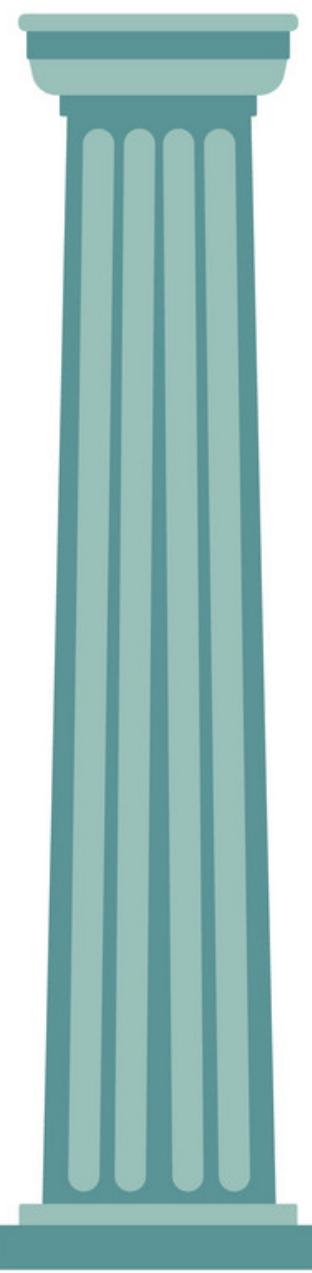


$$\mathfrak{M}(\nu, z_3^2) = \int_{-1}^1 dx C(x\nu, \mu^2 z_3^2) f(x, \mu^2) + \mathcal{O}(z_3^2 \Lambda^2) \quad C(\xi, \mu^2 z_3^2) = e^{i\xi} - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \left[\frac{1+w^2}{1-w} \log \left(z_3^2 \mu^2 \frac{e^{2\gamma_E+1}}{4} \right) + 4 \frac{\log(1-w)}{1-w} - 2(1-w) \right]_+ e^{i\xi w} + \mathcal{O}(\alpha_s^2)$$

This formula allows to relate collinear PDFs to quantities which are computable in lattice QCD simulations, through a factorized expression similar to those relating collinear PDFs to physical cross sections. It can be used in a fitting framework, to extract PDFs from lattice data, performing the same kind of analysis which is usually done when considering experimental data.

Precision calculations for the LHC

CC#2 : Reach the 1% goal by the start of the HL-LHC

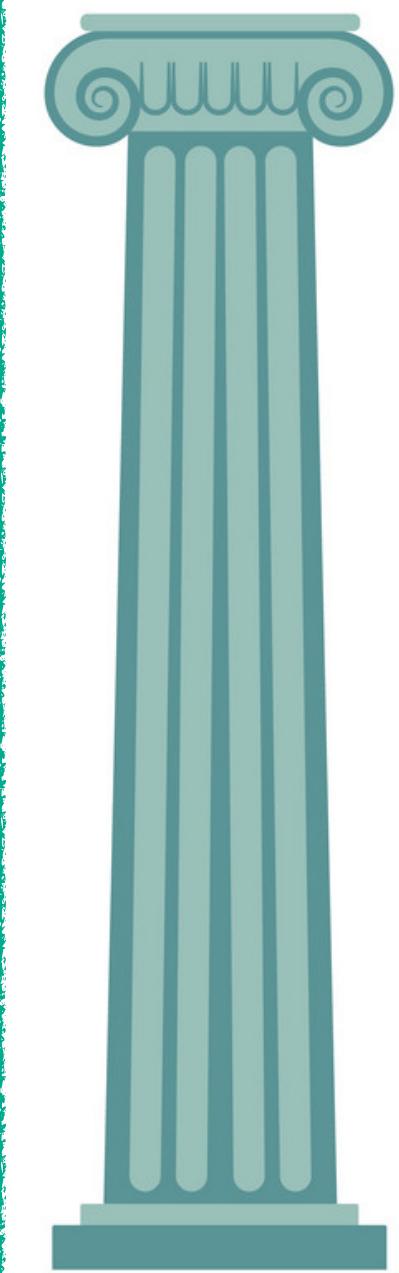


Fixed Order

LO, NLO,...

QCD/EW

- Very fast progress in conceptual as well as technical aspects.
- Tight and consolidated community, with high momentum.
- Considering the status of 20 years ago seems clear that NNLO will be completed and N3LO will start to become available for $2 \rightarrow 2$ (see 3-loop $q\bar{q} \rightarrow \gamma\gamma$ results)
- Mixed QCD-EW being included.

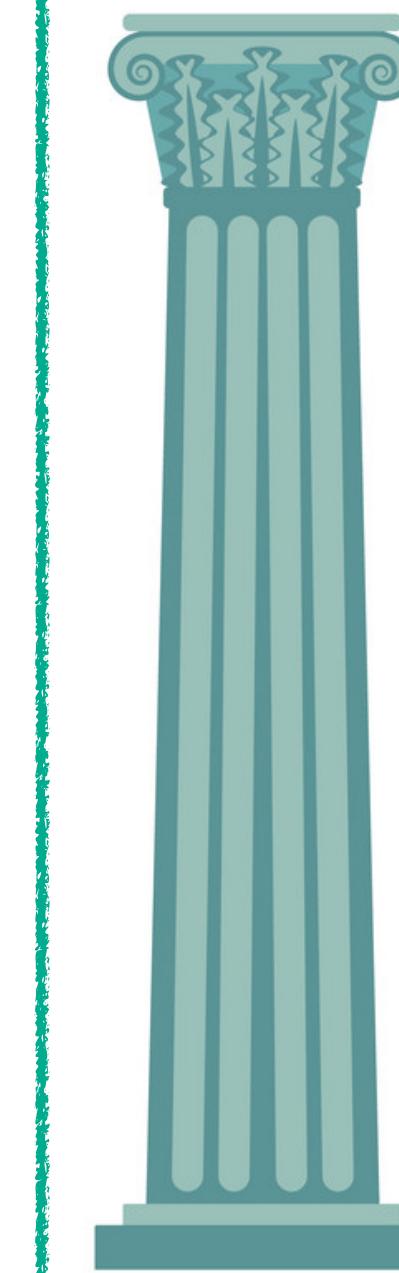


Resum

LL, NLL,...

PS

- A variety of approaches available, both analytical and numerical.
- Analytically historically matching the FO accuracy.
- NNLO+PS will be the new standard. (N3LO+PS already being explored)
- Having a NLL and beyond PS, is being explored now. To be seen.
- Not clear whether one can reach 1%.



PDF's

LO,NLO,..

Fits

- Complete N3LO PDF's evolution not available yet.
- PDF determination from fitting large set of data. Final quality depends on measurements.
- Error budget with many sources. MHO uncertainties yet to be included in the final assessment.
- Reaching 1% will be very challenging.
- Room for a breakthrough from lattice.

Precision calculations for the LHC

What about New Physics?

- Precision predictions for popular (SUSY) BSM scenarios have been made available over the years.
- As the BSM options proliferate, automatic methods to compute at least NLO corrections have been used.
- In any case, the generic rationale is that they are not needed for discovery (don't change the sensitivity) but only in the limit setting.
- Are there significant exceptions?



Searching for new interactions with an EFT

A simple approach

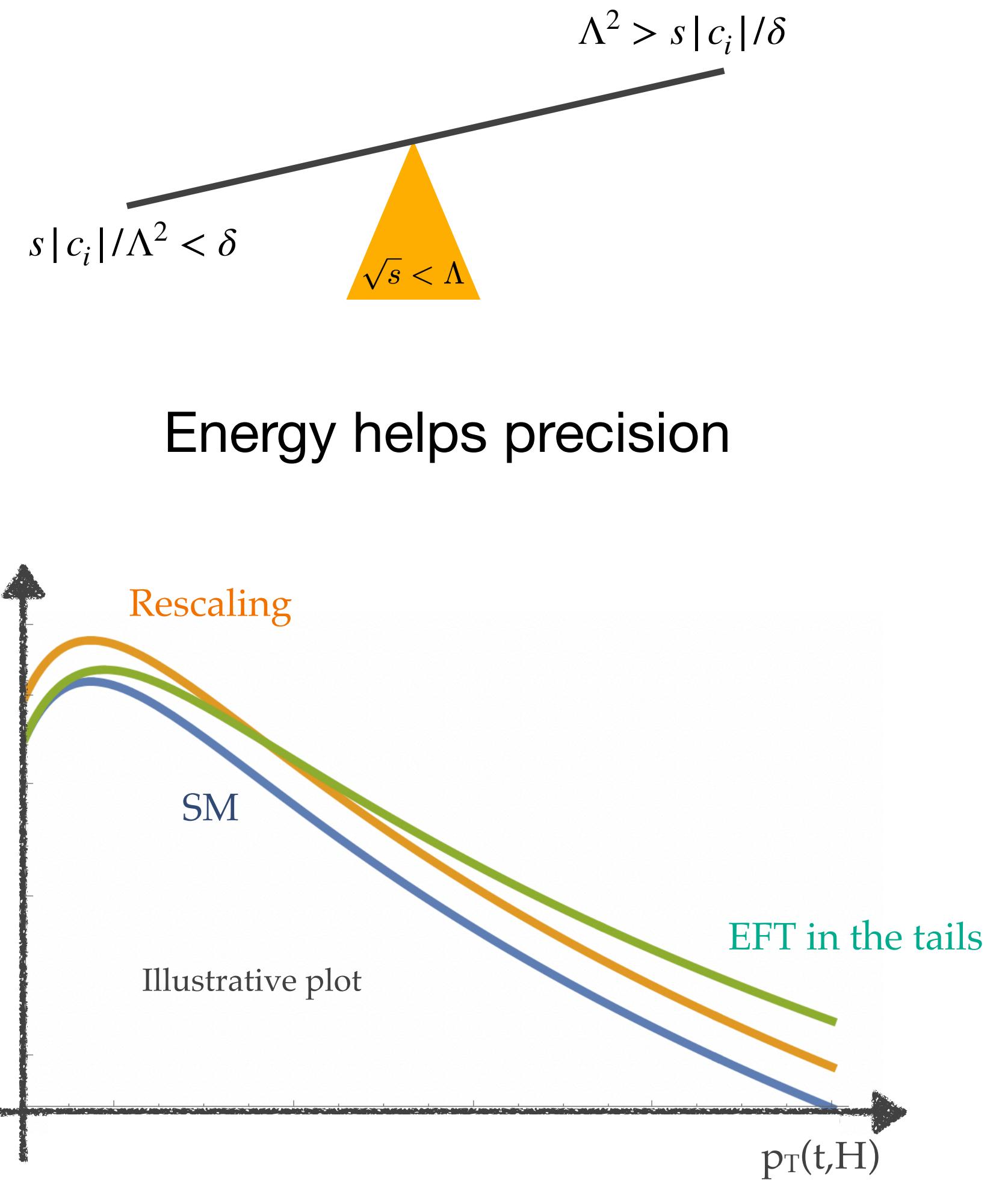
One can satisfy all the previous requirements, by building an EFT on top of the SM that respects the gauge symmetries:

$$\mathcal{L}_{SM}^{(6)} = \mathcal{L}_{SM}^{(4)} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \dots$$

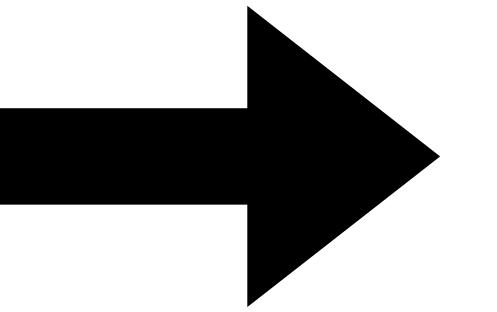
With the “only” assumption that all new states are heavier than energy probed by the experiment $\sqrt{s} < \Lambda$.

The theory is renormalizable order by order in $1/\Lambda$, perturbative computations can be consistently performed at any order, and the **theory is predictive**, i.e., well defined patterns of deviations are allowed, that can be further limited by adding assumptions from the UV. **Operators can lead to larger effects at high energy (for different reasons).**

* Sufficiently weakly interacting states may also exist without spoiling the EFT.



Restyling the SM



SM 1967



SM(EFT) 2020

Searching for new interactions with an EFT

Interpretation needs precision

The master equation of an EFT approach has three key elements:

$$\Delta \text{Obs}_n = \text{Obs}_n^{\text{EXP}} - \text{Obs}_n^{\text{SM}} = \frac{1}{\Lambda^2} \sum_i a_{n,i}^{(6)}(\mu) c_i^{(6)}(\mu) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

The diagram shows a triangle with three vertices representing different components in the master equation:

- Top vertex (red circle):** Obs_n^{SM}
- Bottom-left vertex (blue circle):** $\text{Obs}_n^{\text{EXP}}$
- Bottom-right vertex (green circle):** $\frac{1}{\Lambda^2} \sum_i a_{n,i}^{(6)}(\mu) c_i^{(6)}(\mu)$

Arrows point from each vertex to its corresponding text description below the triangle:

- An arrow from the red circle points to the text: "Most precise SM predictions for observables: NLO, NNLO, N3LO..."
- An arrow from the blue circle points to the text: "Most precise/accurate experimental measurements with uncertainties and correlations"
- An arrow from the green circle points to the text: "Most precise EFT predictions"

Searching for new interactions with an EFT

Interpretation needs precision

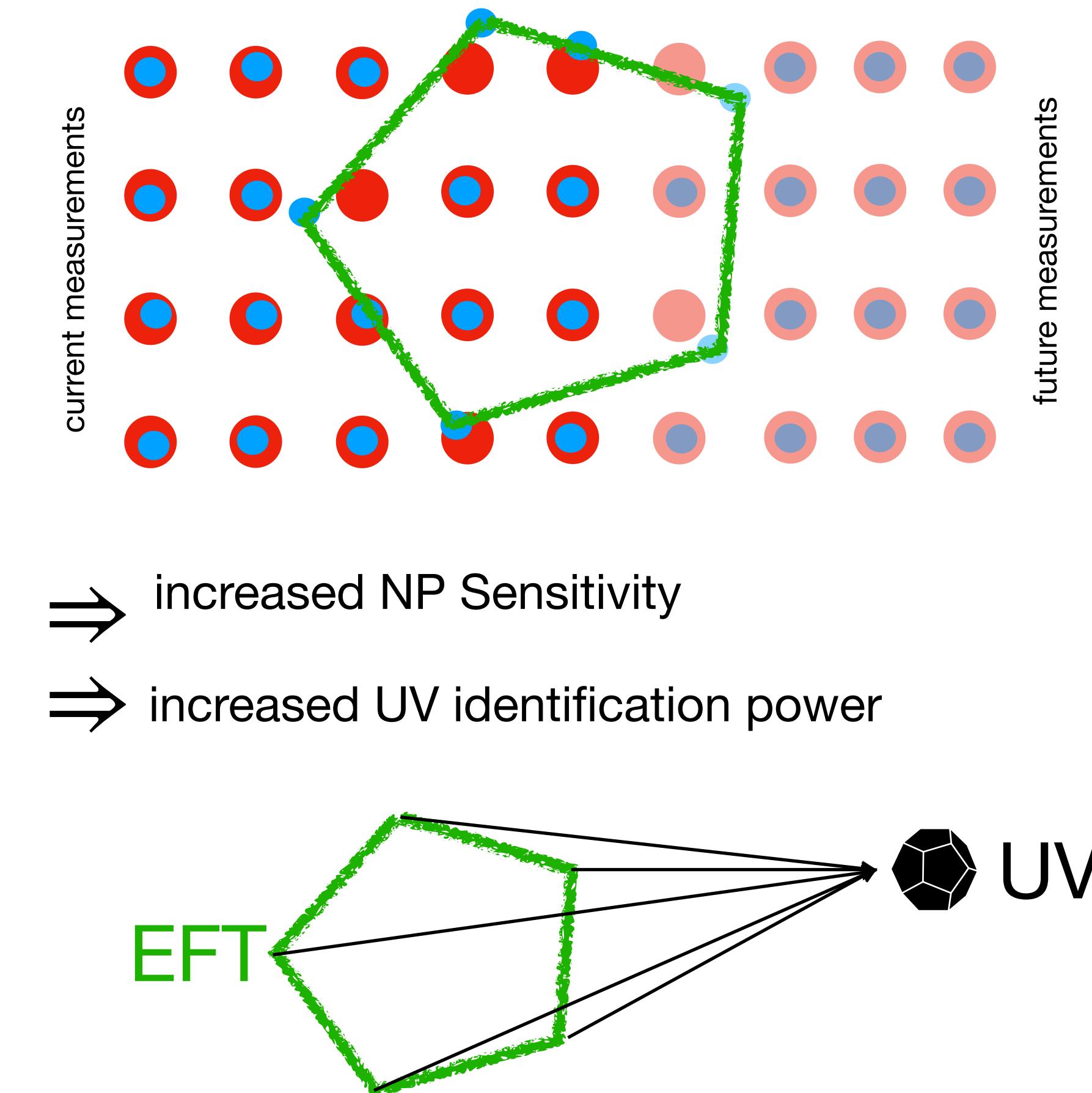
The master equation of an EFT approach has three key elements:

$$\Delta \text{Obs}_n = \text{Obs}_n^{\text{EXP}} - \text{Obs}_n^{\text{SM}} = \frac{1}{\Lambda^2} \sum_i a_{n,i}^{(6)}(\mu) c_i^{(6)}(\mu) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Most precise EFT predictions

Most precise SM predictions for observables:
NLO, NNLO, N3LO...

Most precise/accurate experimental measurements with uncertainties
and correlations

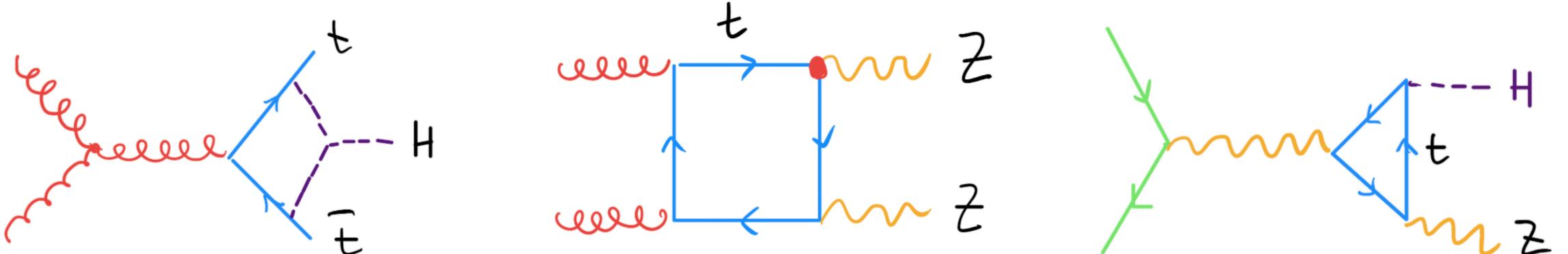


Searching for new interactions with an EFT

Progress in SMEFT at 1-loop level

1-loop accuracy allows:

- Unveil the SMEFT structure (mixing)
- K-factors (accuracy)
- Scale uncertainties (precision)
- Exploit loop sensitivity:



“same strategy” as in SM@dim4

RGE

- Anomalous dimension matrix [[Jenkins, Manohar and Trott, 2013,2014,2014](#)]

Production

- $p\bar{p} \rightarrow jj$ (4F) [[Gao, Li, Wang, Zhu, Yuan, 2011](#)]
- $p\bar{p} \rightarrow tt$ (4F) [[Shao, Li, Wang, Gao, Zhang, Zhu, 2011](#)]
- $p\bar{p} \rightarrow VV$ [[Dixon, Kunszt, Signer, 1999](#)] [[Melia, Nason, Röntsch, Zanderighi, 2011](#)]
[[Baglio, Dawson, Lewis, 2017,2018,2019](#)][[Chiesa et al., 2018](#)]
- top FCNCs [[Degrande, FM, Wang, Zhang, 2014](#)] [[Durieux, FM, Zhang, 2014](#)]
- $p\bar{p} \rightarrow tt$ (chromo) [[Franzosi, Zhang, 2015](#)]
- $p\bar{p} \rightarrow tj$ [[Zhang, 2016](#)] [[de Beurs, Laenen, Vreeswijk, Vryonidou, 2018](#)]
- $p\bar{p} \rightarrow ttZ$ [[Röntsch and Schulze, 2015](#)] [[Bylund, FM, Tsinikos, Vryonidou, Zhang, 2016](#)]
- $p\bar{p} \rightarrow ttH$ [[FM, Vryonidou, Zhang, 2016](#)]
- $p\bar{p} \rightarrow HV, Hjj$ [[Greljo, Isidori, Lindert, Marzocca, 2015](#)][[Degrande, Fuks, Mawatari, Mimasu, Sanz, 2016](#)], [[Alioli, Dekens, Girard, Mereghetti, 2018](#)]
- $p\bar{p} \rightarrow H$ [[Grazzini, Ilnicka, Spira, Wiesemann, 2016](#)] [[Deutschmann, Duhr, FM, Vryonidou, 2017](#)]
- $p\bar{p} \rightarrow tZj, tHj$ [[Degrande, FM, Mimasu, Vryonidou, Zhang, 2018](#)]
- $p\bar{p} \rightarrow$ jets [[Hirschi, FM, Tsinikos, Vryonidou, 2018](#)]
- $p\bar{p} \rightarrow VVV$ [[Degrande, Durieux, FM, Mimasu, Vryonidou, Zhang, 20xx](#)]
- $gg \rightarrow ZH, Hj, HH$ [[Bylund, FM, Tsinikos, Vryonidou, Zhang, 2016](#)]
- Higgs self-couplings [[McCullough, 2014](#)][[Degrassi, Giardino, FM, Pagani, Shivaji, Zhao, 2016-2018](#)][[Borowka et al. 2019](#)][[FM, Pagani, Zhao, 2019](#)]
- EW loops in tt [[Kuhn et al., 1305.5773](#)], [[Martini 1911.11244](#)]
- EW top loops in Higgs & EW [[Vryonidou, Zhang, 2018](#)][[Durieux, Gu, Vryonidou, Zhang, 2018](#)]
[[Boselli et al. 2019](#)]
- Drell-Yan (EW corrections) [[Dawson and Giardino, 2021](#)]

Decay

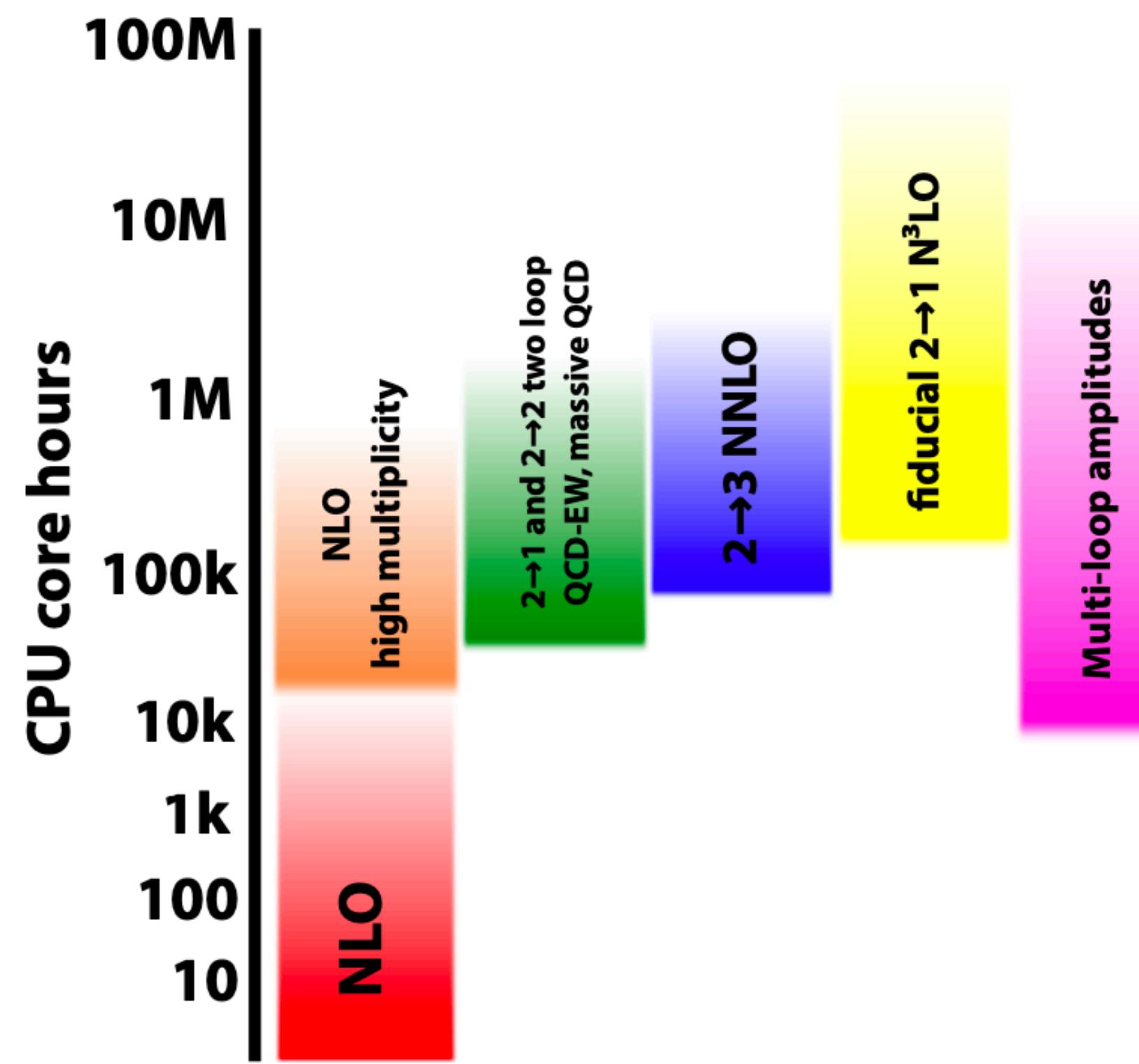
- Top [[Zhang, 2014](#)] [[Boughezal, Chen, Petriello, Wiegand, 2019](#)]
- $h \rightarrow VV$ [[Hartmann, Trott, 2015](#)] [[Ghezzi, Gomez-Ambrosio, Passarino, Uccirati, 2015, 2015](#)]
[[Dawson, Giardino, 2018,2018](#)][[Dedes, et al., 2018](#)][[Dedes, Suxho, Trifyllis, 2019](#)]
- $h \rightarrow ff$ [[Gauld, Pecjak, Scott, 2016](#)] [[Cullen, Pecjak, Scott, 2019](#)][[Cullen, Pecjak, 2020](#)]
- Z, W [[Hartmann, Shepherd, Trott, 2016](#)] [[Dawson, Ismail, Giardino, 2018,2018,2019](#)]

EWPO

- EWPO [[Zhang, Greiner, Willenbrock '12](#)] [[Dawson, Giardino, 2020](#)]

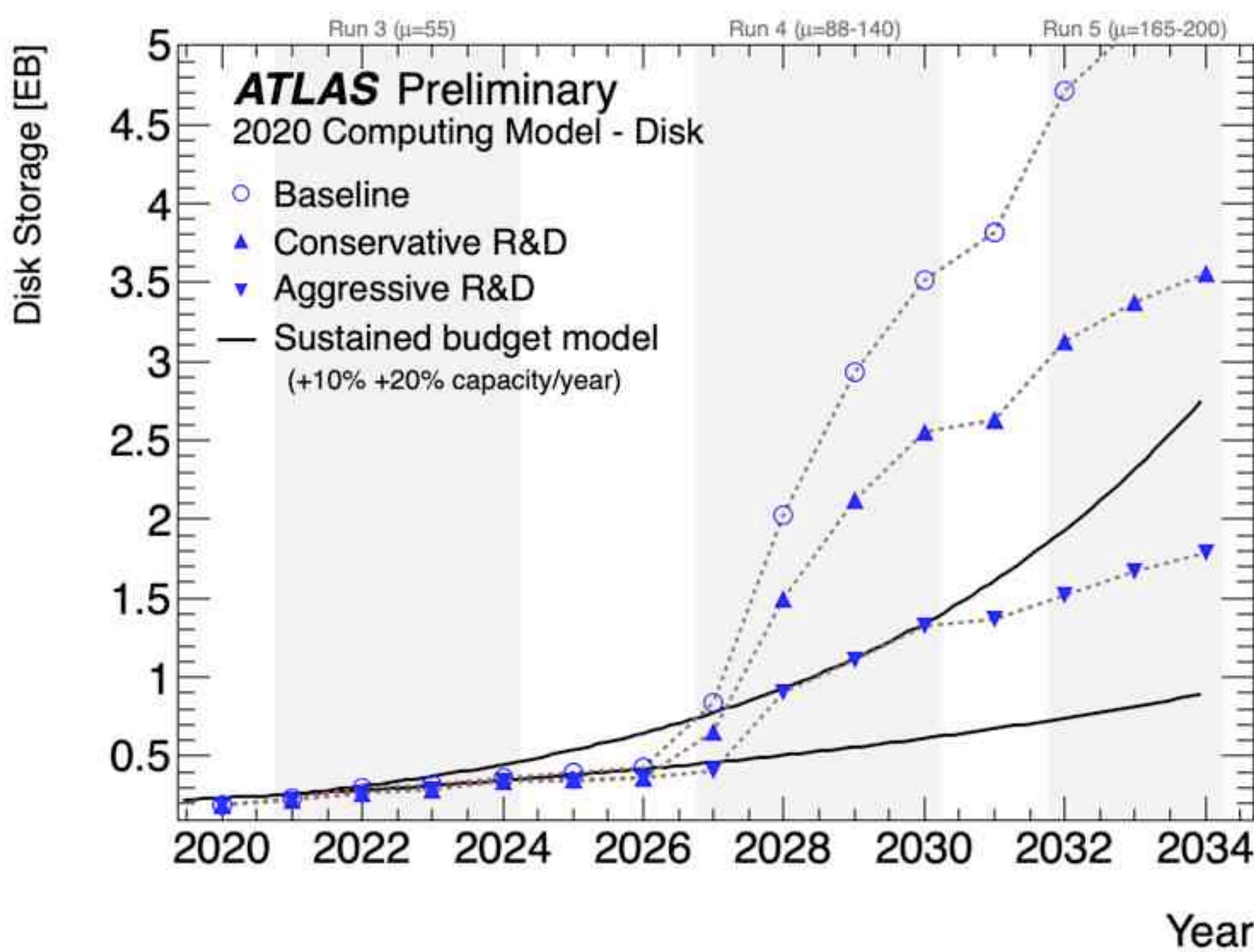
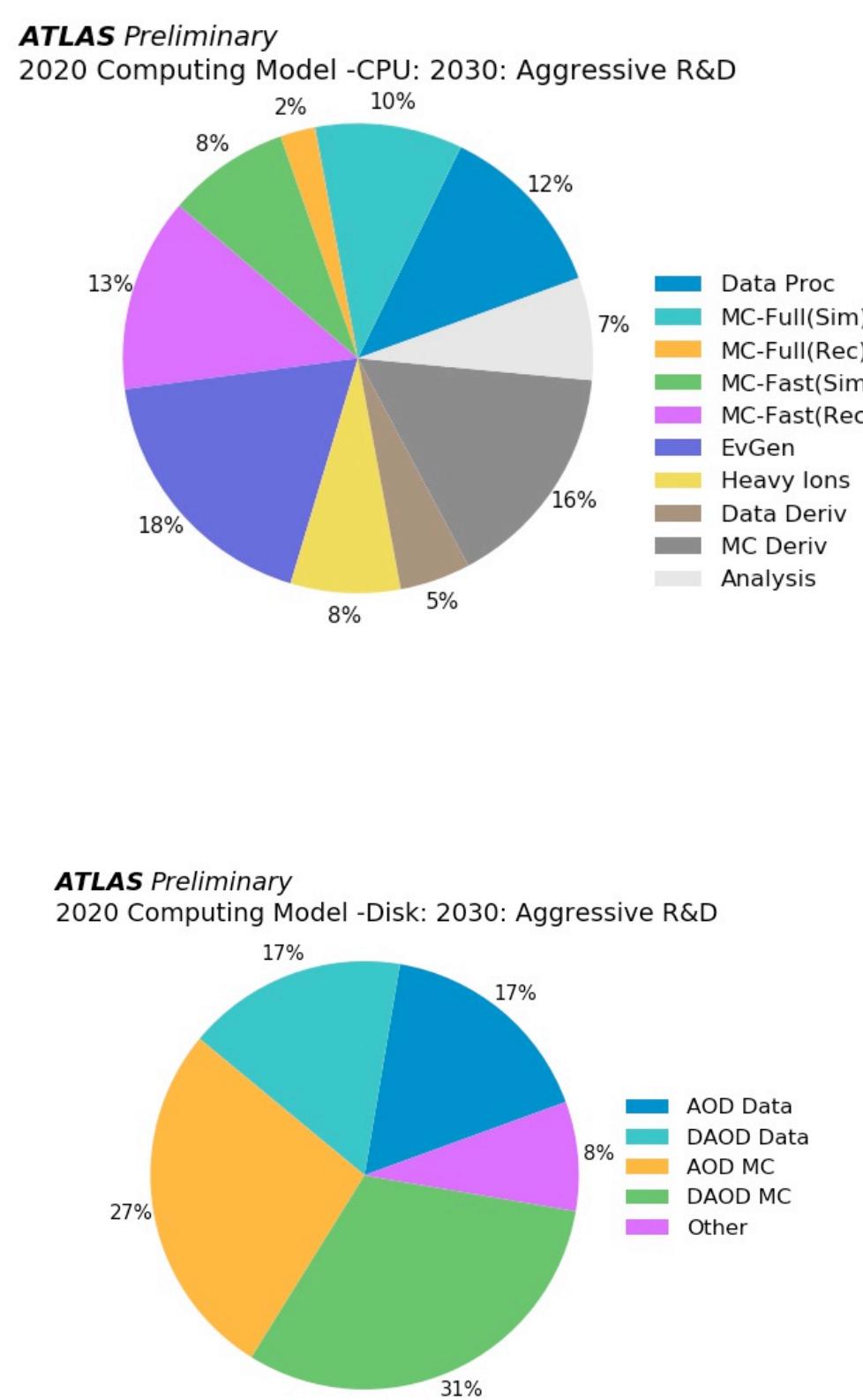
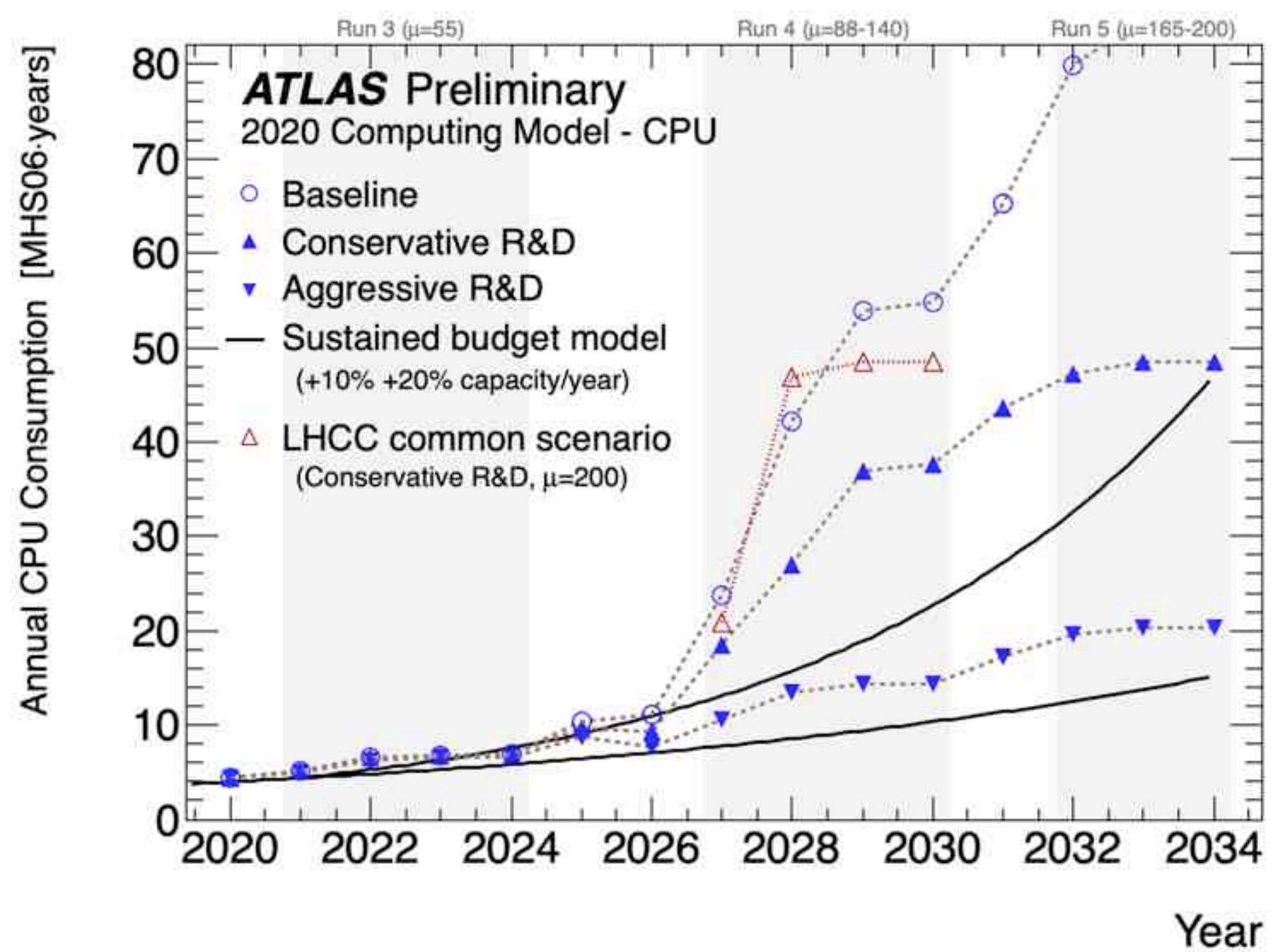
Precision calculations for the LHC

Computing needs



Higher precision \Rightarrow more computing and (memory) storage

Precision calculations for the LHC Computing needs

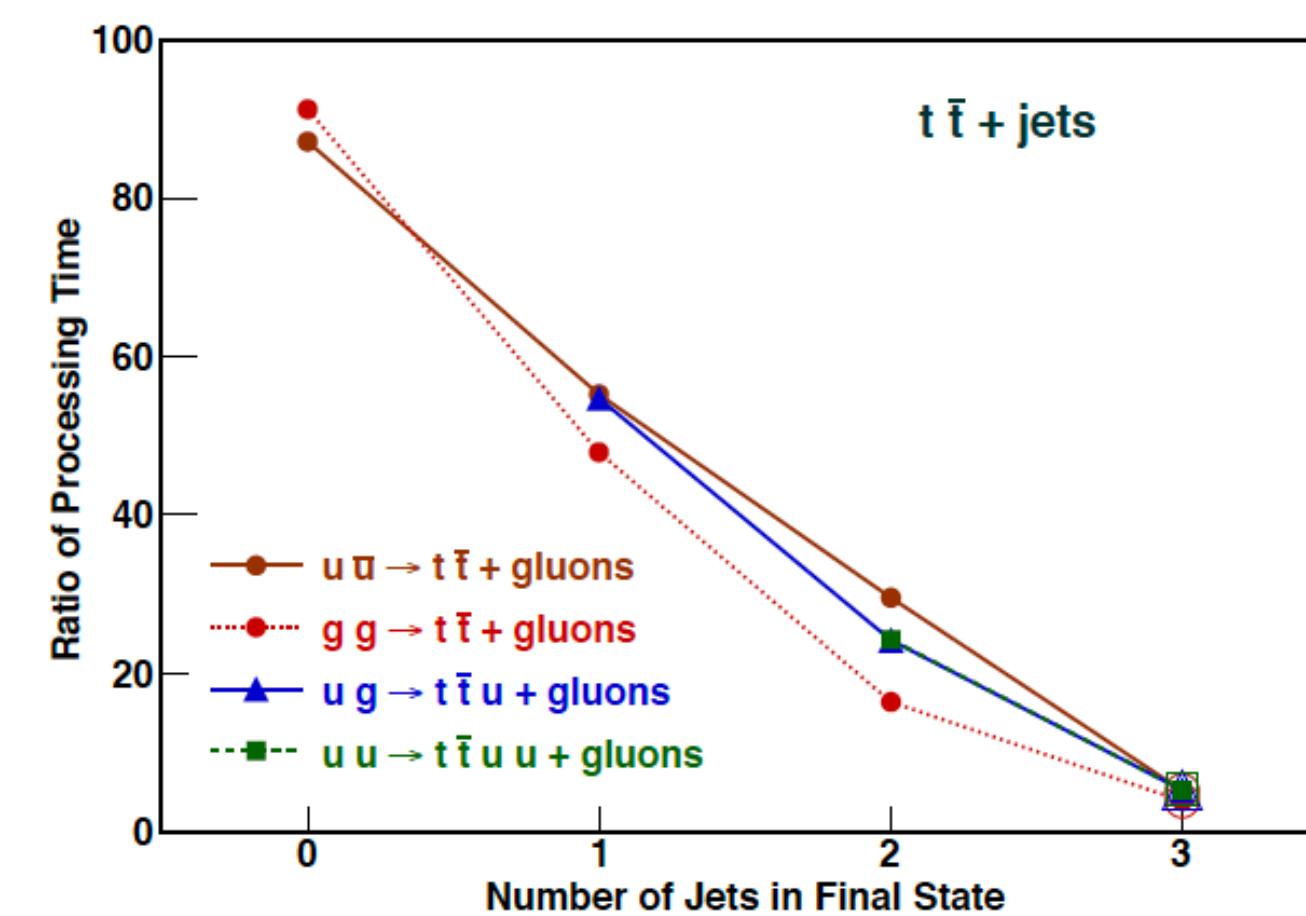


CC#3: develop tools that can provide all the necessary predictions/simulations within the planned computing and storage resources.

Computational opportunities

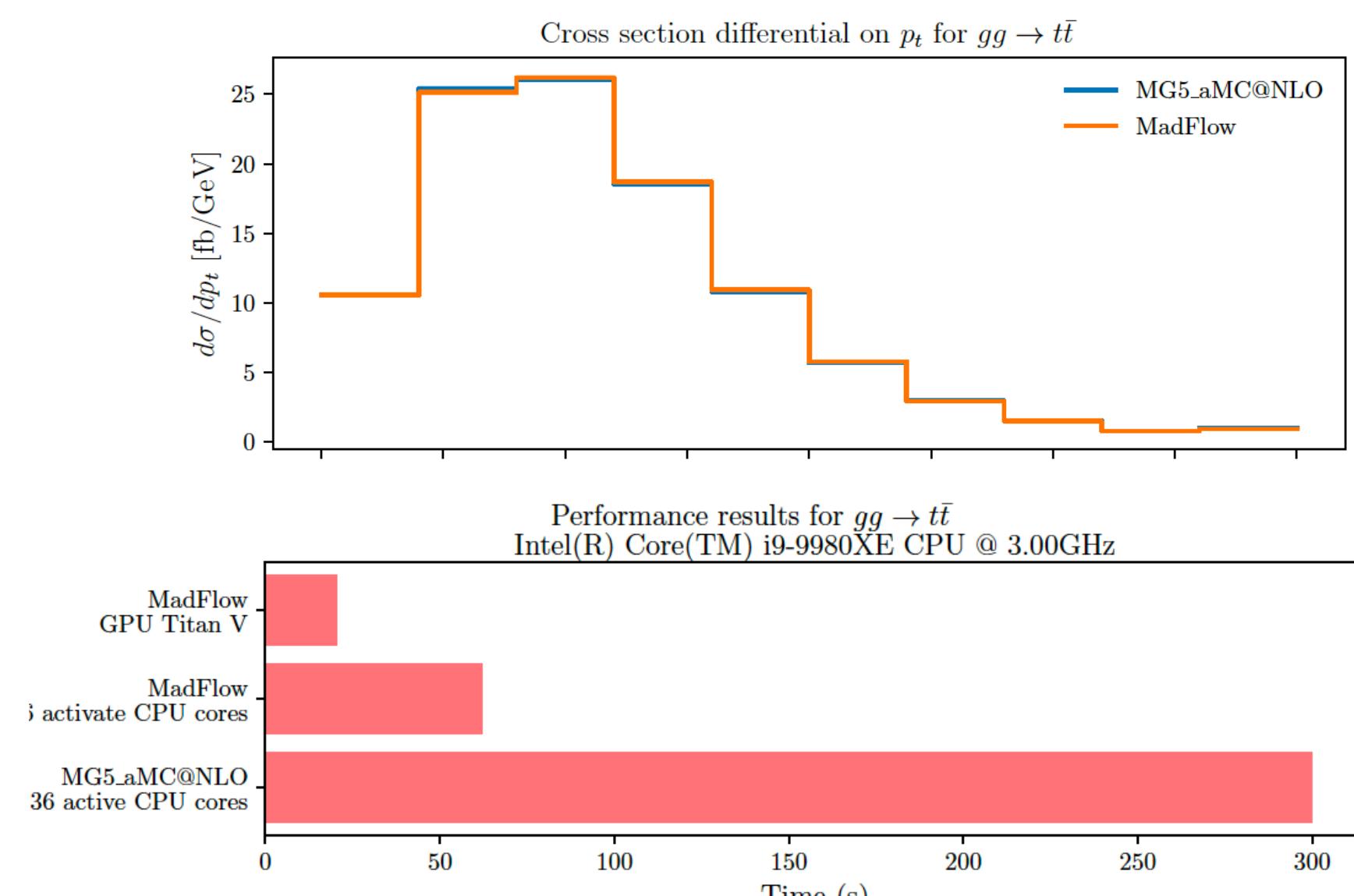
New architectures

MadGraph on GPUs 2013



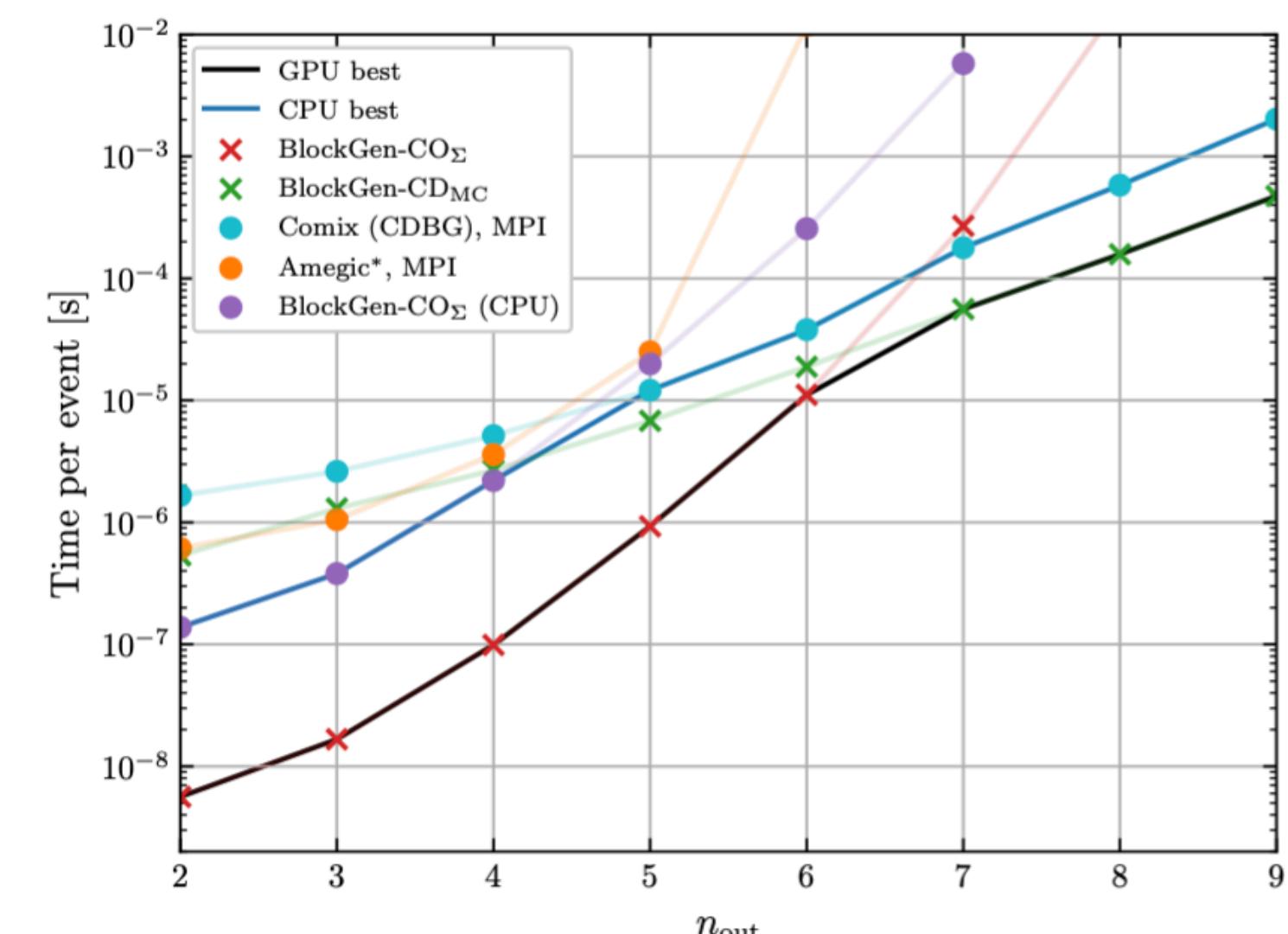
[Hagiwara et al. , 1305.0708]

MadGraph



[S. Carrazza et al. , 2105.10529]

Comix/Amegic on GPUs



[E. Bothmann, et al. 2106.06507]

Proof of principle implementation based on CUDA and first GPUs. Memory constraints, large color matrices \rightarrow huge gains but scaling with # extra partons bad...

Modern approach based on new architectures,

Computational opportunities

Machine Learning techniques

[A survey of machine learning-based physics event generation](#)
 [Y. Alanazi, et al. 2106.00643]

[Understanding Event-Generation Networks via Uncertainties](#)

[M. Bellagente et al 2104.04543]

[Phase Space Sampling and Inference from Weighted Events with Autoregressive Flows](#)

[B. Stienen et al. , 2011.13445]

[i-flow: High-dimensional Integration and Sampling with Normalizing Flows](#)

[Christina Gao et al. 2001.05486]

[How to GAN Event Unweighting](#)

[M. Backes et al. : 2012.07873]

[Generative Networks for LHC events](#)

[Anja Butter and Plehn 2008.08558]

[Invertible Networks or Partons to Detector and Back Again](#)

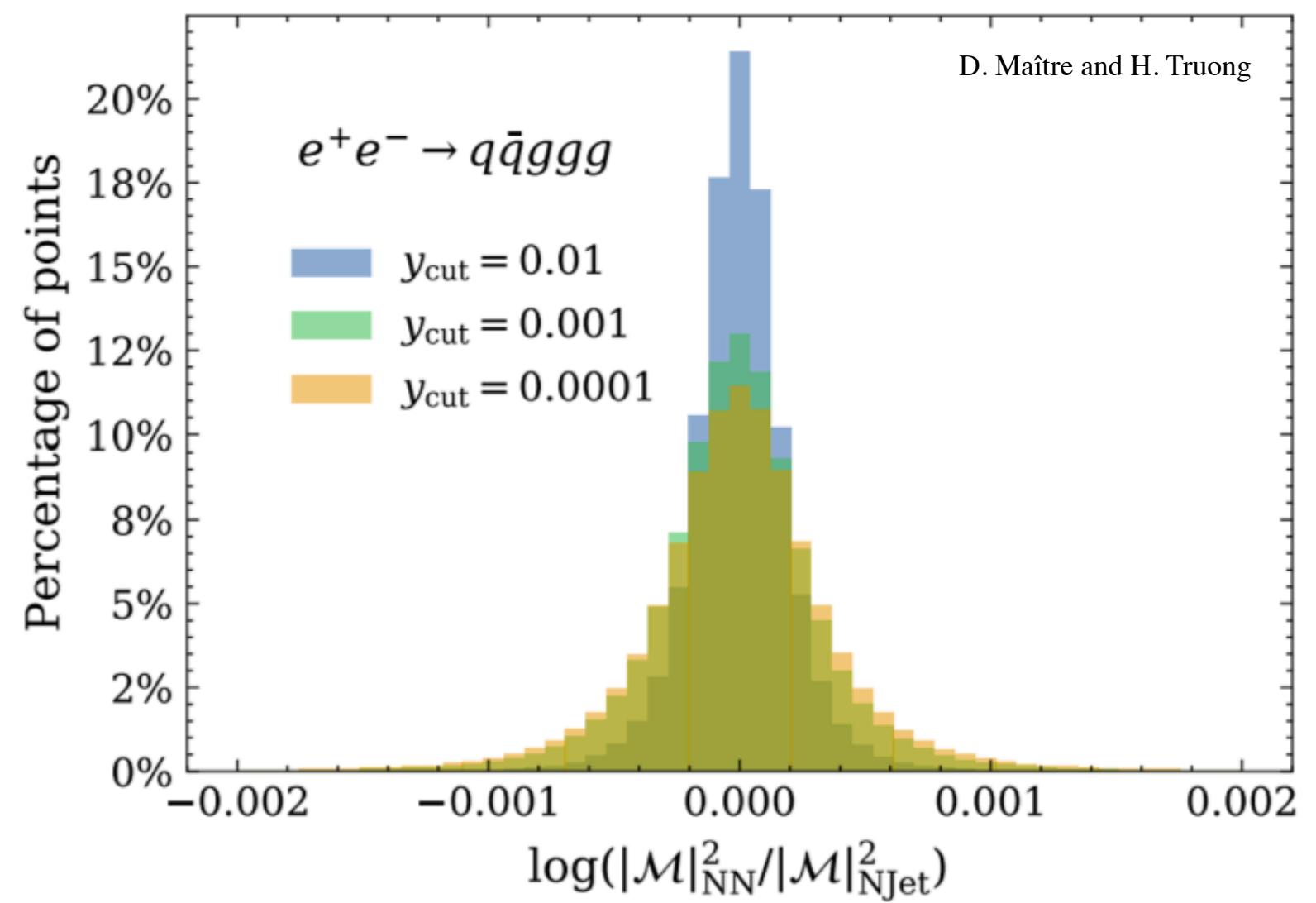
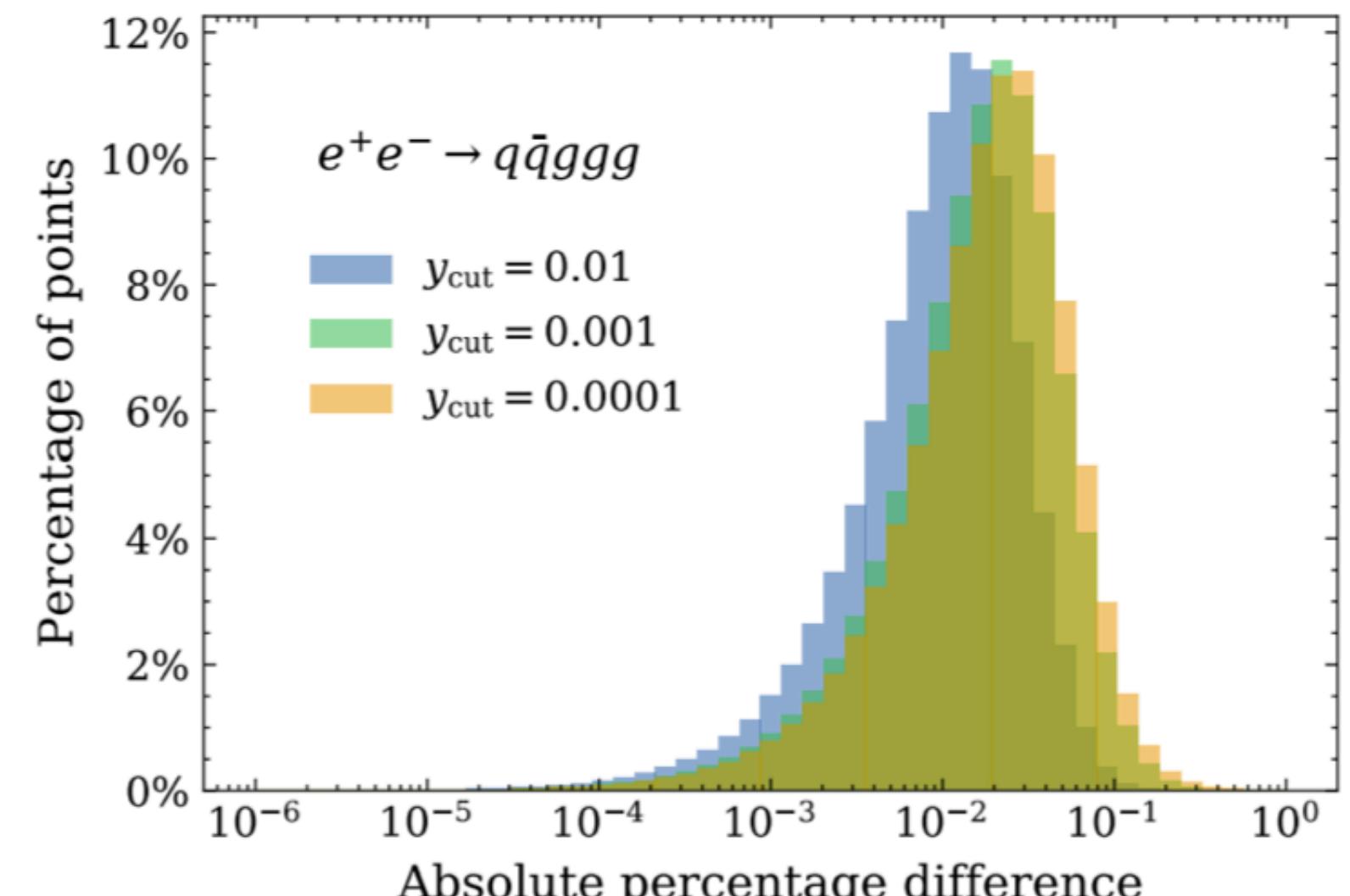
[M. Bellagente et al. e-Print: 2006.06685]

[How to GAN away Detector Effects](#)

[M. Bellagente et al, 1912.00477]

[How to GAN LHC Events](#)

Anja Butter et al. 1907.03764 [hep-ph]



Impressive progress in the exploration of different methods and in identifying the most relevant questions in last few years!

- Can the ML-MC go beyond the statistical precision of the training event samples?
- Can they faithfully reproduce the physics?
- Can they provide new physics insights?

Computational opportunities

Quantum Computing

Growing interest in quantum computations for HEP:

[Quantum Algorithm for High Energy Physics Simulations](#)
[C. W. Bauer et al. 1904.03196]

[Quantum Algorithms for Jet Clustering](#)
Annie Y. Wei et al. 1908.08949 [hep-ph]

[Towards a quantum computing algorithm for helicity amplitudes and parton showers](#)
Khadeejah Bepari et al. 2010.00046 [hep-ph]

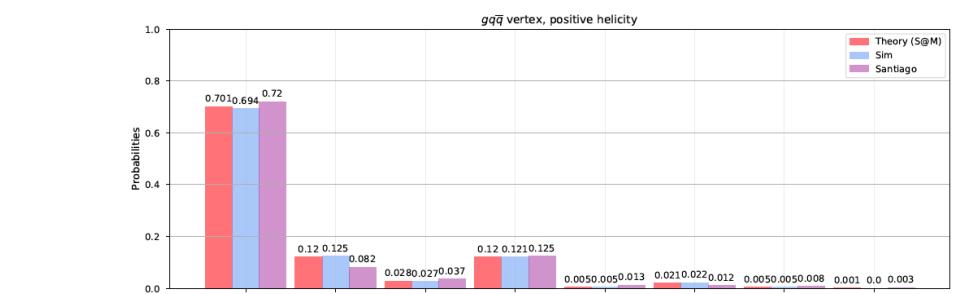
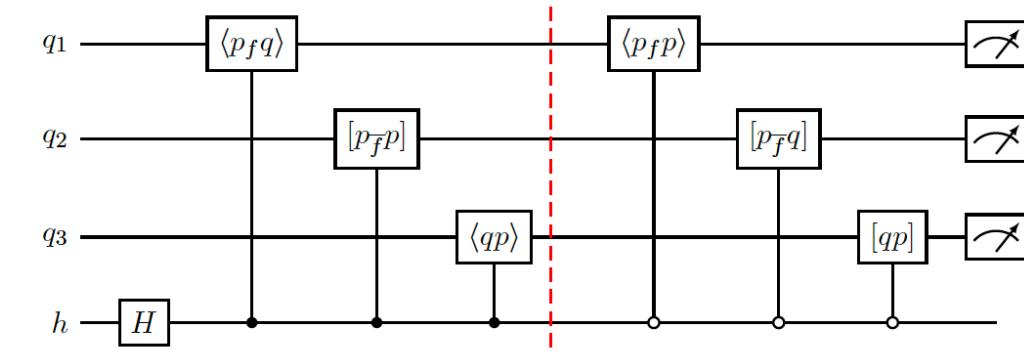
[Determining the proton content with a quantum computer](#)
Adrián Pérez-Salinas et al. 2011.13934

[Simulating collider physics on quantum computers using effective field theories](#)
C. W. Bauer et al. 2102.05044 [hep-ph]

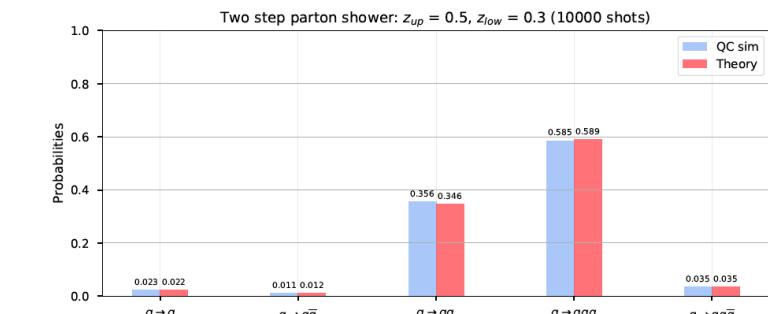
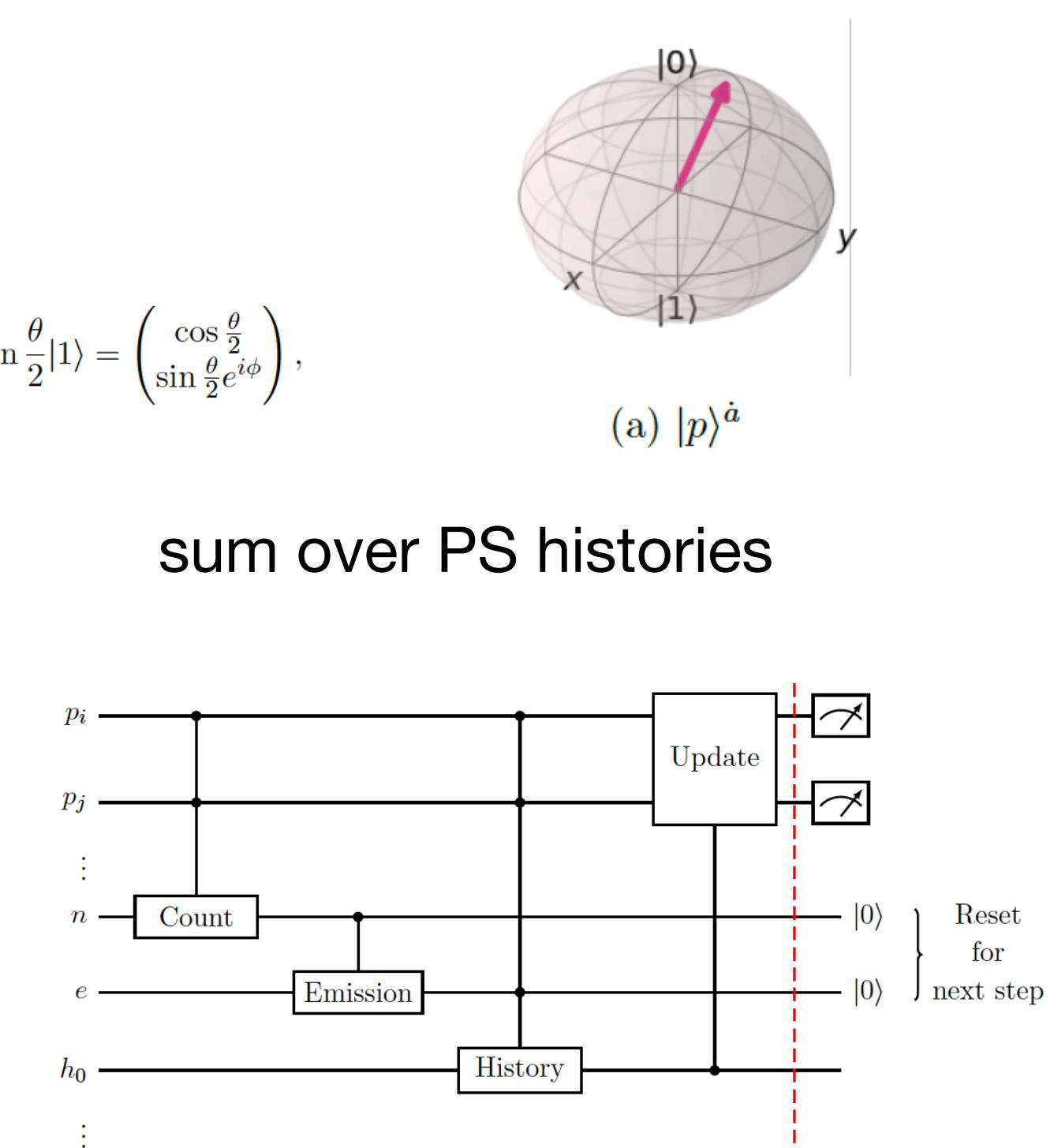
[Quantum algorithm for Feynman loop integrals](#)
Selomit Ramírez-Uribe et al. 2105.08703

sum over helicities

$$\mathcal{M}_+ = -\sqrt{2} \frac{\langle p_f q \rangle [p_{\bar{f}} p]}{\langle q p \rangle}, \quad \mathcal{M}_- = -\sqrt{2} \frac{\langle p_f p \rangle [p_{\bar{f}} q]}{\langle q p \rangle}.$$



sum over PS histories



Computational opportunities

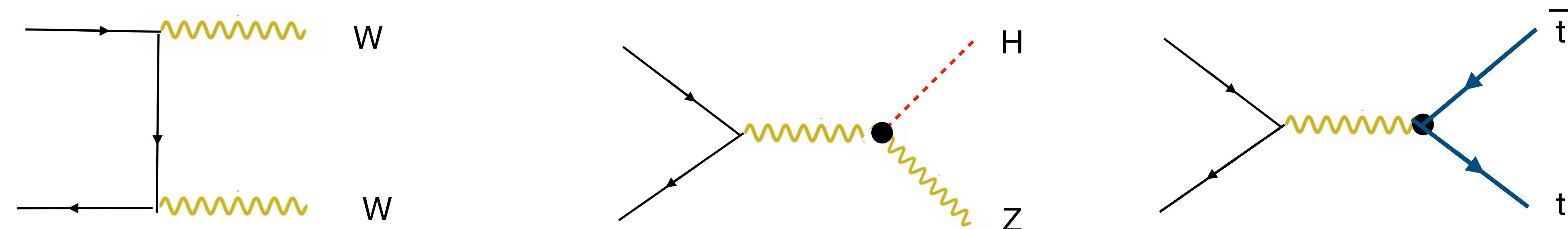
Considerations

- Serious computational bottleneck ahead of us for HL-LHC
- Many really new ideas being explored
- Having an ambitious objective to estimate the gap might help:

Achieve real-time event simulation

Precision calculations for EW.H.T factories

The workhorses



Known at NLO in EW with W decays. In order to determine m_W at 1 MeV needs to be known at the subpermill level. NNLO EW computation involves many scales. In addition an EFT treatment of the W threshold is necessary.

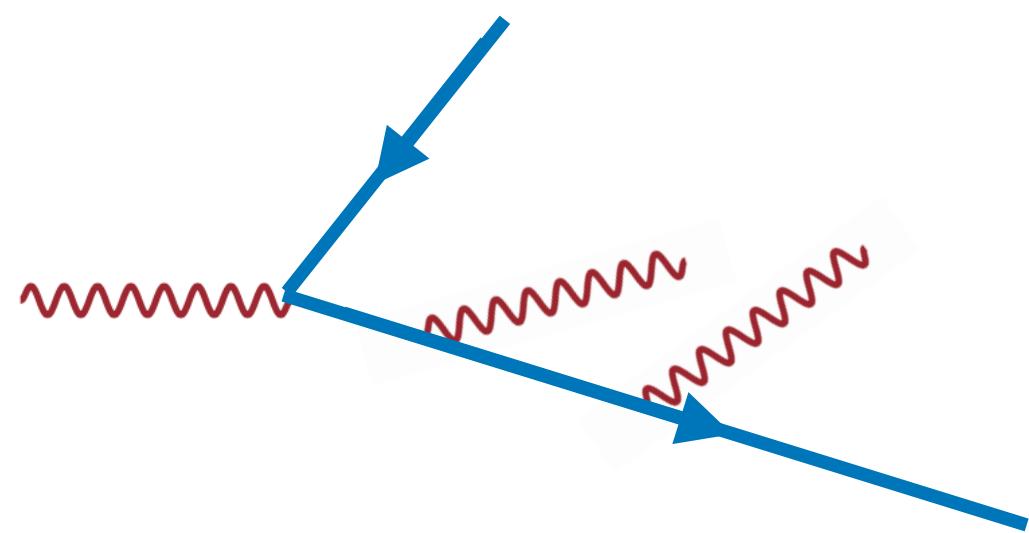
Workhorse for H studies. Known at NLO in EW with Z decays. NNLO correct. Gives access to trilinear and top-Yukawa at one loop and quadrilinear Higgs self-couplings and others at two loops.

Known at N3LO in the NRQCD EFT approach at threshold for top mass and width determination. NLO QCD corrections for the $2 \rightarrow 6$ known. NNLO EW corrections are not known.

In addition ISR effects, collinear and soft need to be included.

Precision calculations for EW.H.T factories

QED showers



In QED the charges are scalars. No flows! $-Qi * Qj$ can be negative emission of a photon is independent from the others and the dipole does not change. Interleaving with the much more probable QCD radiation in the case of quarks is a challenge.

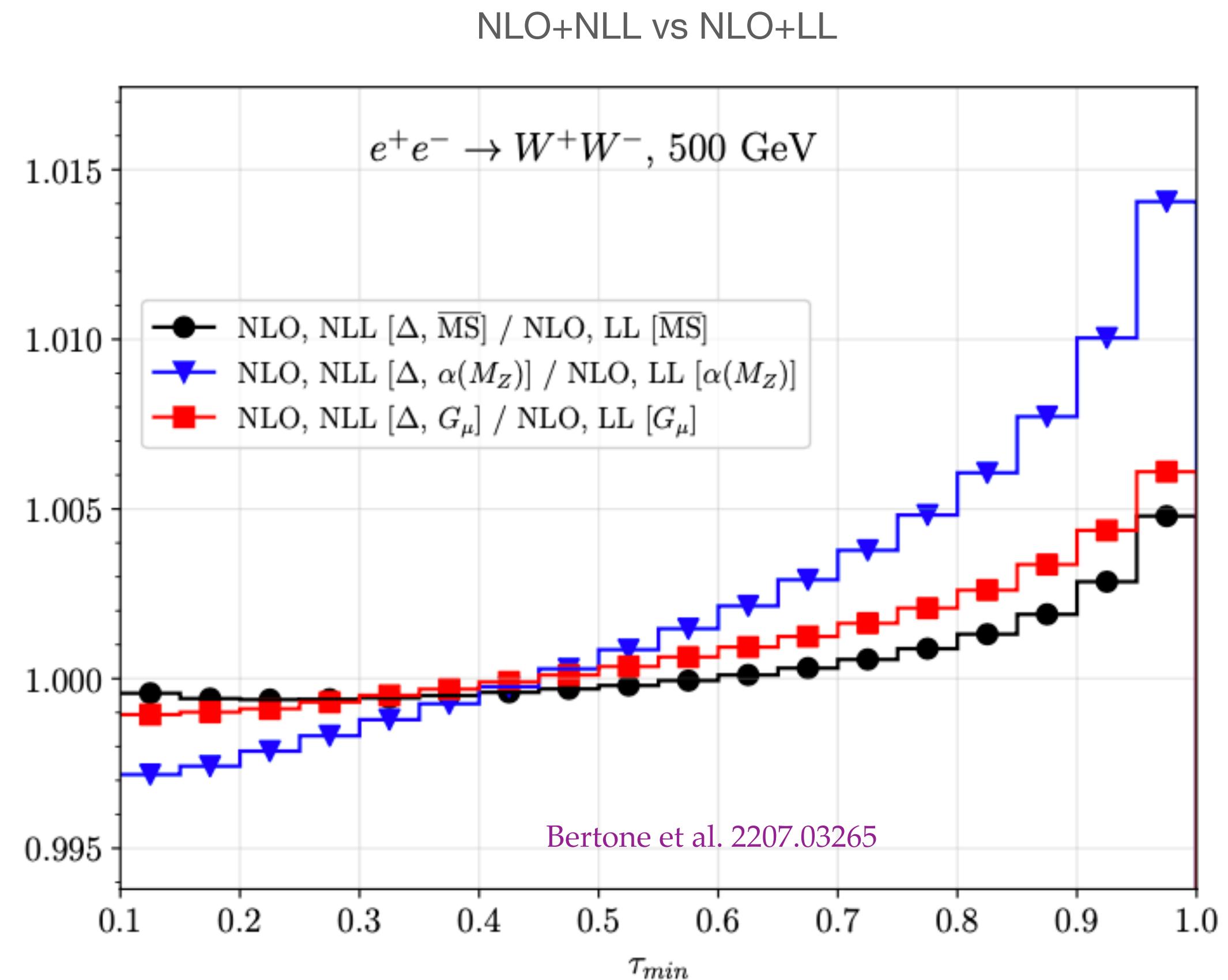
However, YFS Soft resummation provides a way to resum soft contributions

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{1}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \right] (2\pi)^4 \delta^4 \left(\sum_{\text{in}} q_{\text{in}} - \sum_{\text{out}} q_{\text{out}} - \sum_{i=1}^{n_\gamma} k_i \right) \left| \sum_{n_\gamma^V=0}^{\infty} \mathcal{M}_{n_\gamma}^{n_\gamma^V + \frac{1}{2}n_\gamma} \right|^2.$$

→

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{S}(k_j)} + \sum_{j,k=1}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{\tilde{S}(k_j) \tilde{S}(k_k)} + \dots \right)$$

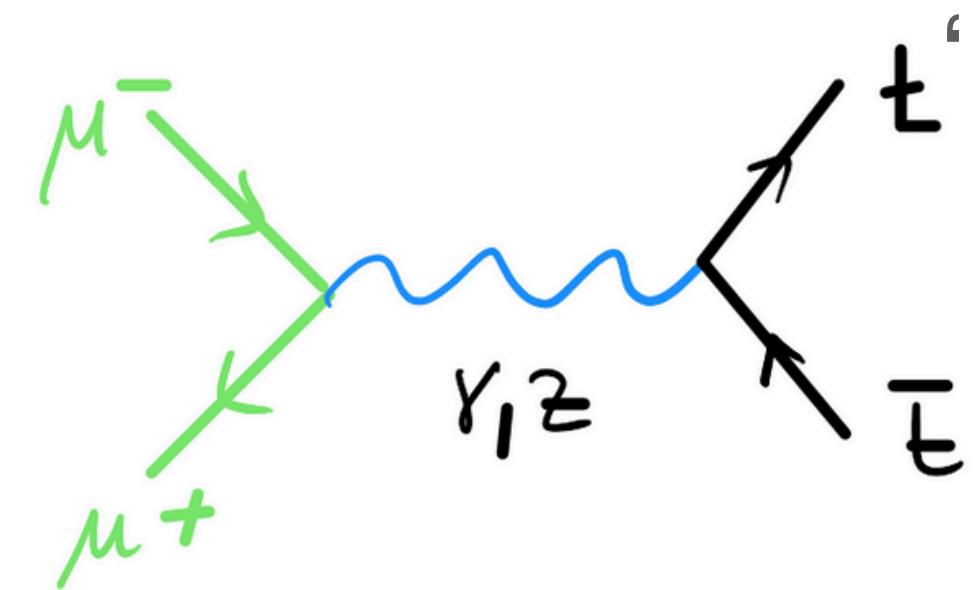
Collinear effects captured through the residuals. Improvements necessary to also have γ to fermions splitting included at order a^2 .



Multi-TeV lepton colliders

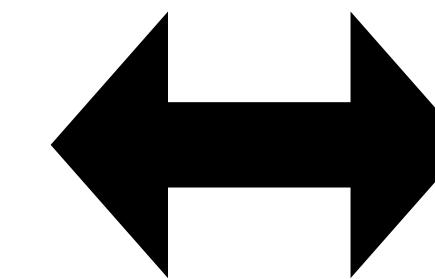
The essentials

O(10) TeV lepton-lepton collider energy allows to have **two colliders in one**:

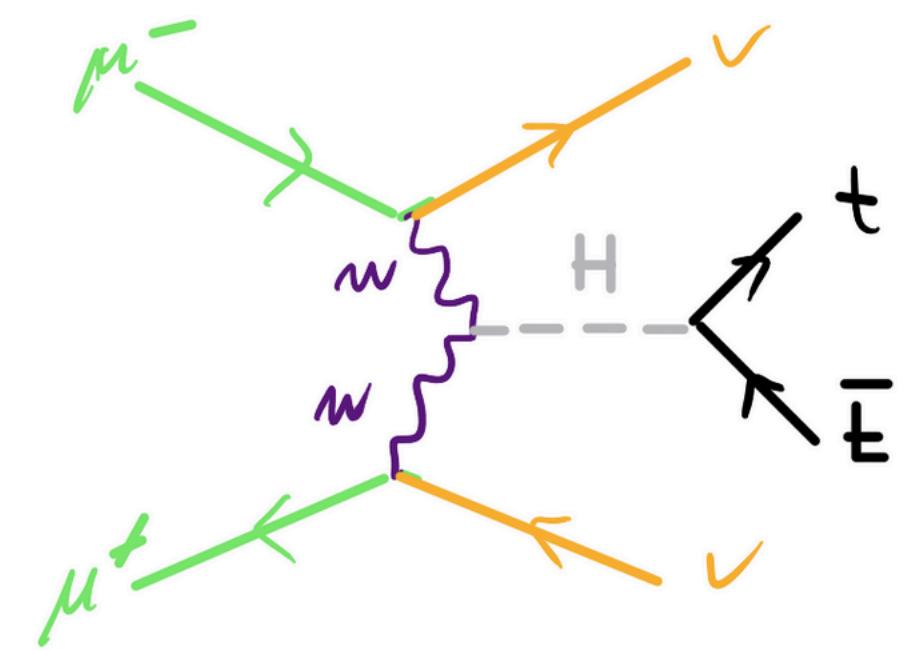


Discovery

$$\sigma_s \sim \frac{1}{s}$$



$$\sigma_s \sim \frac{1}{M^2} \log^n \frac{s}{M}$$

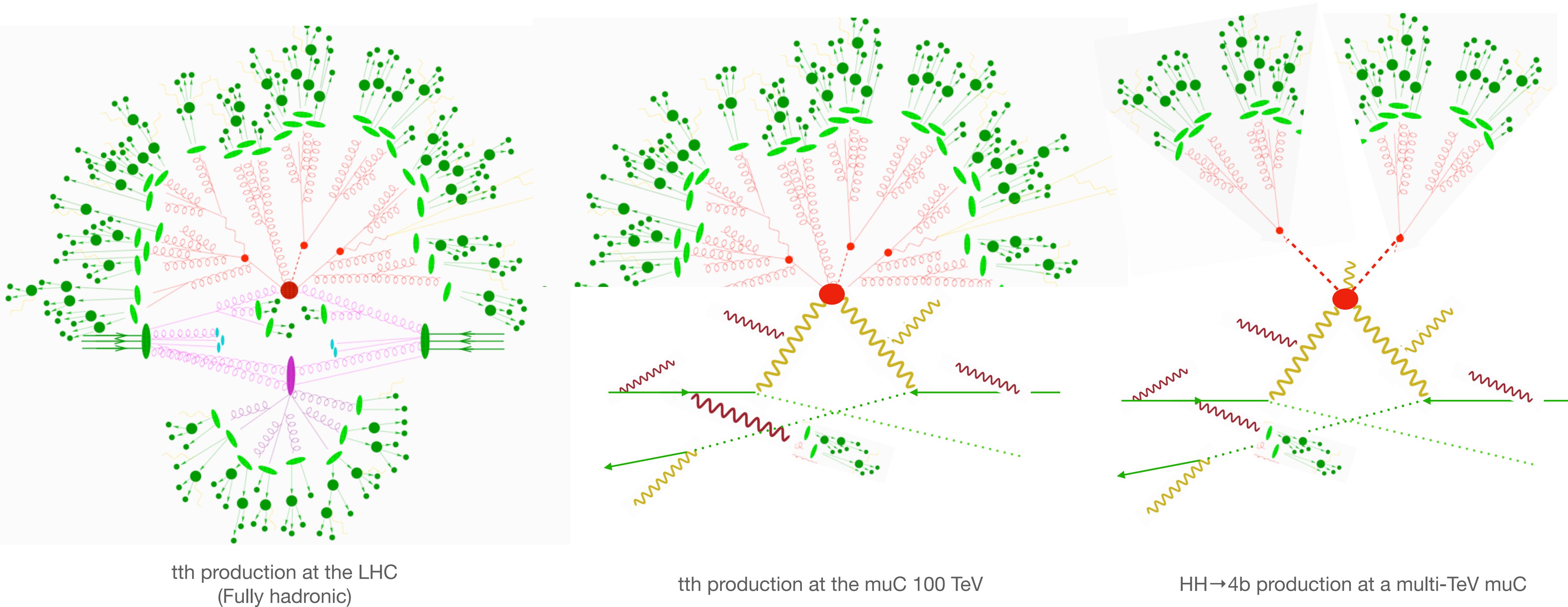


Measurements

A completely new regime opening for a multi-TeV muon collider

Different physics being probed in the two channels

How events look in a multi-TeV lepton collider?



$t\bar{t}$ production at the LHC
(Fully hadronic)

$t\bar{t}$ production at the muC 100 TeV

$HH \rightarrow 4b$ production at a multi-TeV muC

In a muon collider gluons and quarks first appear at scales of order 100 GeV in the decays of W, Z, H (from either initial state or final state radiation) or from photon splitting. Multijet final states are of EW origin.

Precision calculations for multi-TeV lepton colliders

EW resummation

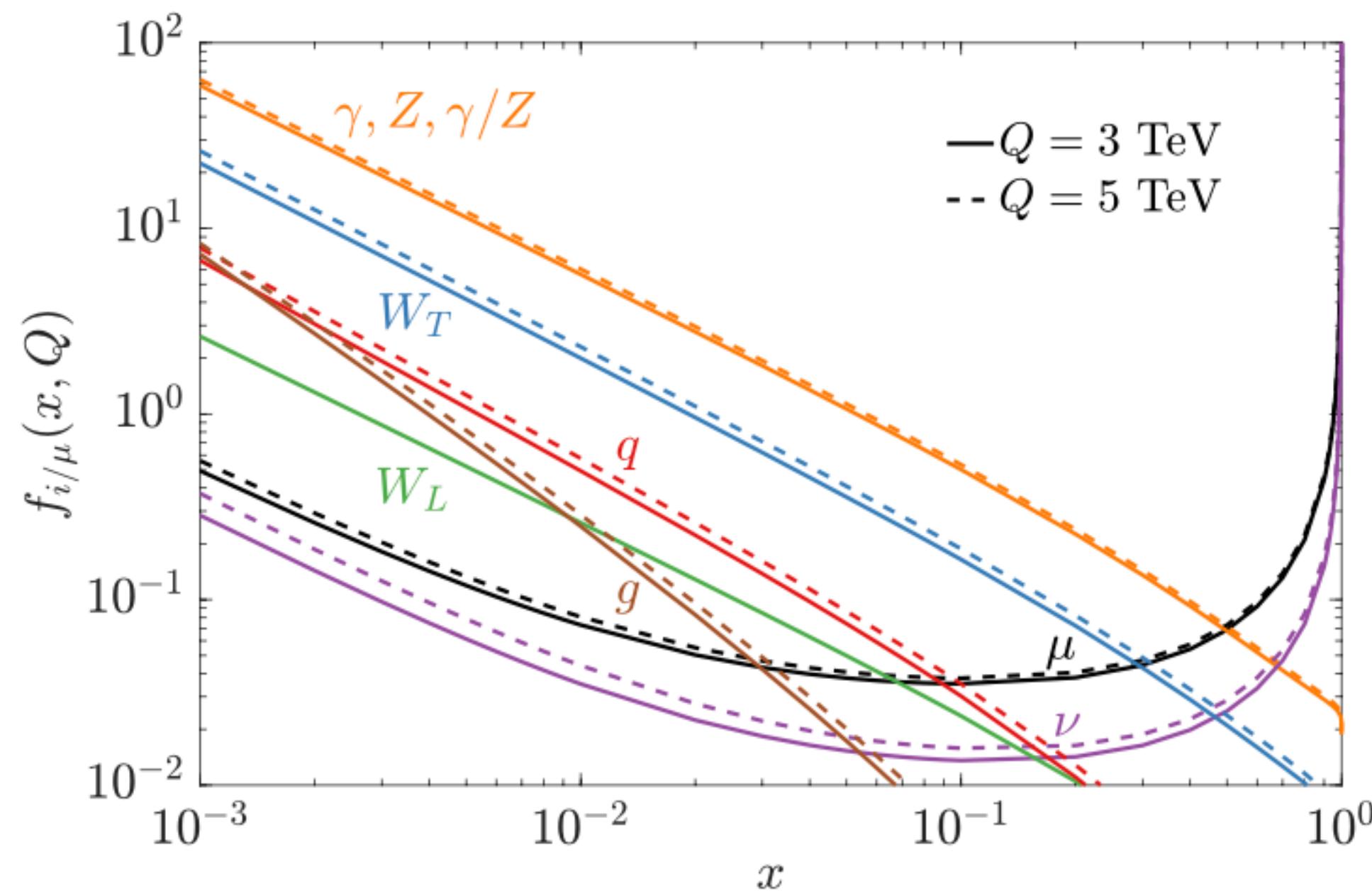
$$f_B(z, \mu^2) = \sum_A \int_z^1 \frac{d\xi}{\xi} f_A(\xi) \int_{m^2}^{\mu^2} dz d\mathcal{P}_{A \rightarrow B+C}(z/\xi, k_T^2)$$

$$\frac{\partial f_B(z, \mu^2)}{\partial \mu^2} = \sum_A \int_z^1 \frac{d\xi}{\xi} \frac{d\mathcal{P}_{A \rightarrow B+C}(z/\xi, \mu^2)}{dz dk_T^2} f_A(\xi, \mu^2)$$

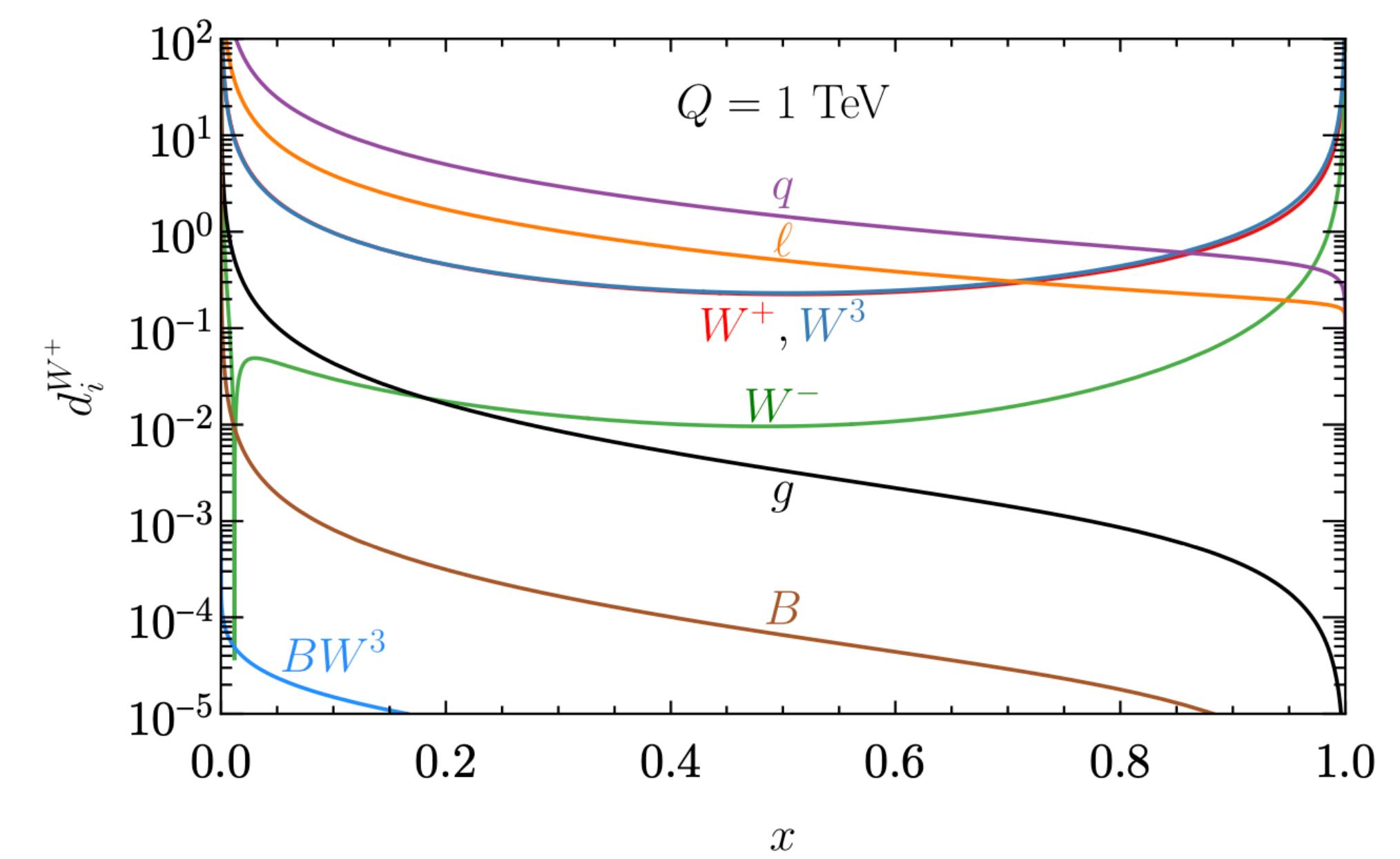
$$\Delta_A(t) = \exp \left[- \sum_B \int_{t_0}^t dz \mathcal{P}_{A \rightarrow B+C}(z) \right],$$

$$f_A(x, t) = \Delta_A(t) f_A(x, t_0) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \mathcal{P}_{A \rightarrow B+C}(z) f_A(x/z, t')$$

Han, Ma, Xie arXiv:2007.14300v4. 2103.09844



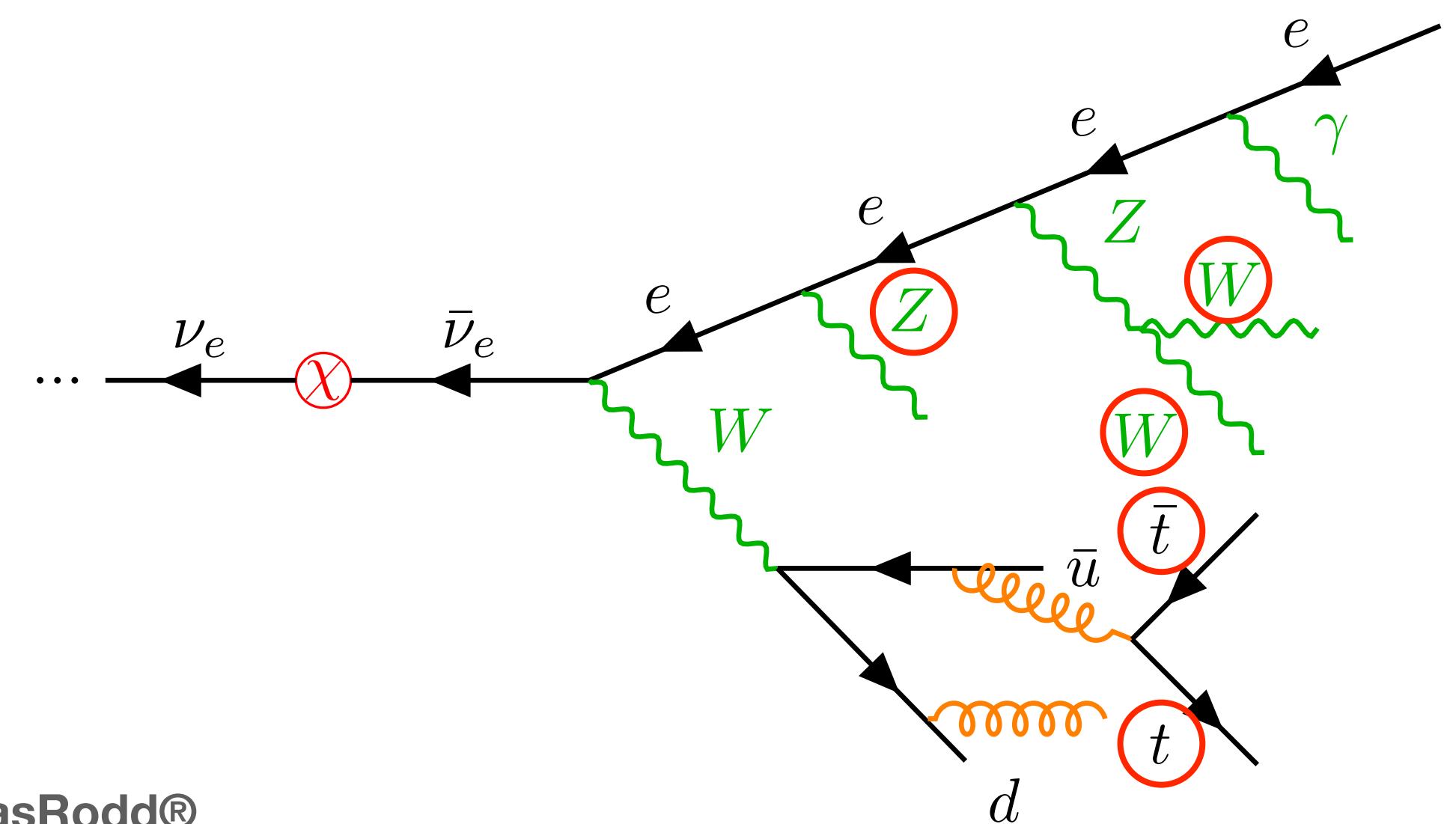
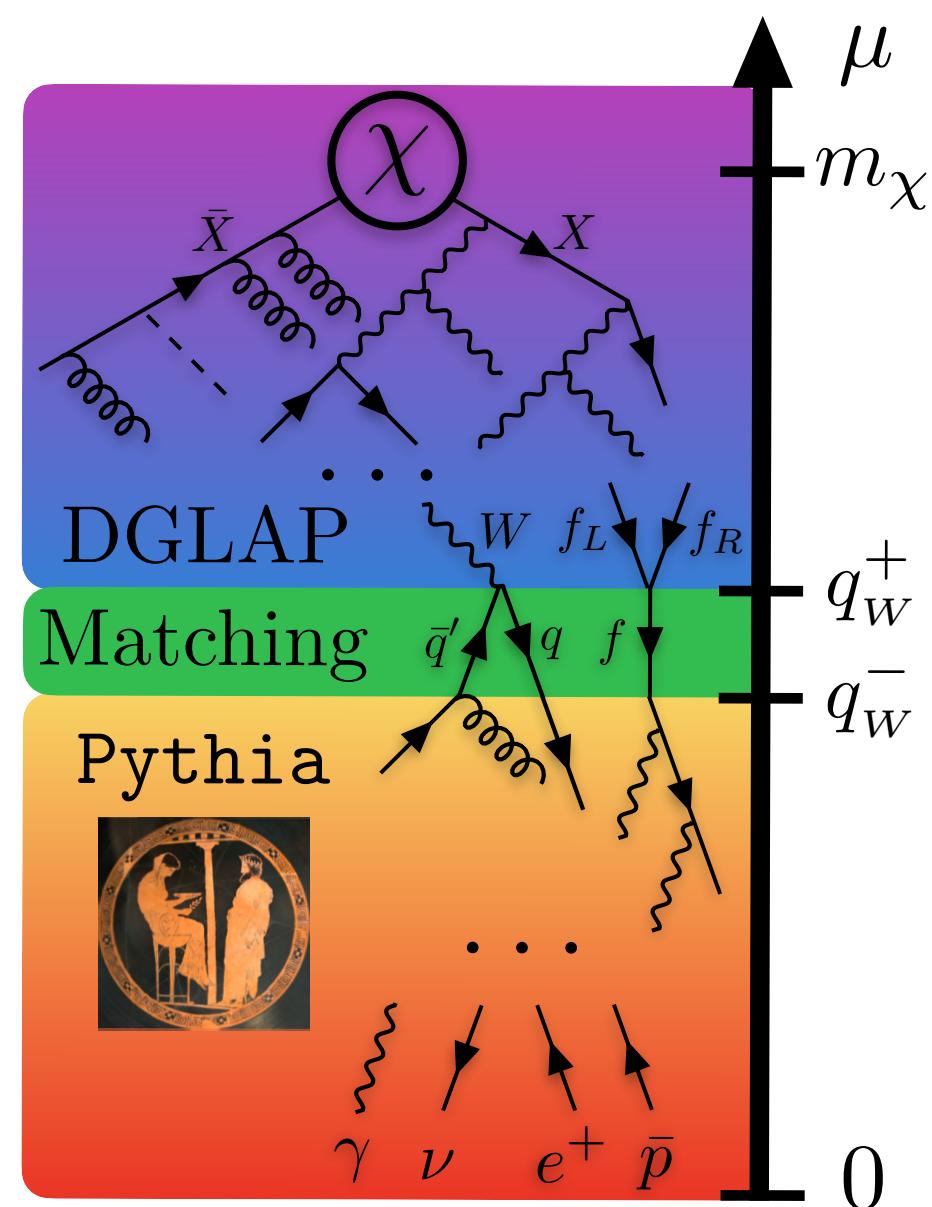
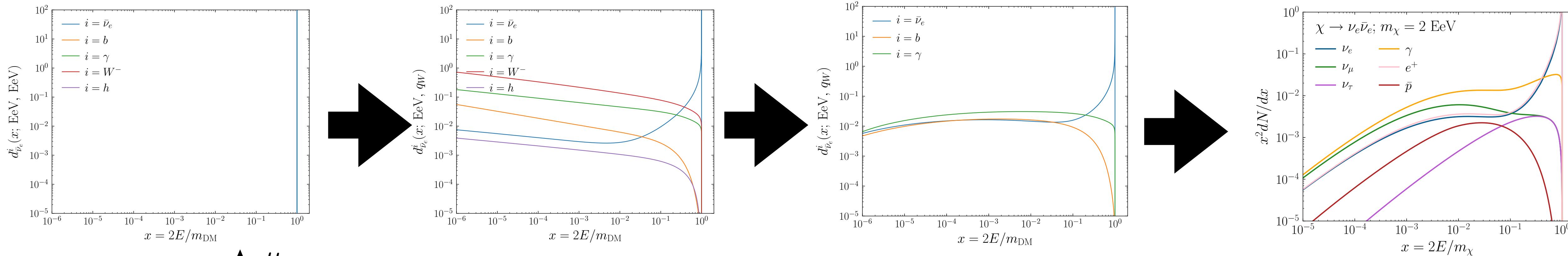
Han, Ma, Xie rXiv:2203.11129v1



Some comments here

Precision calculations for multi-TeV lepton colliders EW showers

At very high energies, $E \gg v$, $SU(2) \times U(1)$ is restored and evolution through EW radiation will take place. The non-abelian nature of $SU(2)$ will make a shower look more like QCD. Once the scales are down to $\sim v$ EWSB effects will start to become important again.



Evolution (EW double logs and polarisation):

- [Christiansen, Sjostrand 1401.5238]
- [Christiansen, Prestel 1510.01517]
- [Chen, Han, Tweedie 1611.00788]
- [Manohar, Waalewijn 1802.08687]
- [Bauer, Provasoli, Webber 1806.10157]
- [Bauer, Webber 1808.08831]
- [Kleiss, Verheyen, 2002.09248]
- [Bauer, Rodd, Webber 2007.15001]
- [Masouminia, Richardson, 2108.10817]
- [Brooks, Skands, Verheyen 2108.10786v2]

Computational challenges for high-energy colliders

Summary

- #1: Determine the SM parameters in terms of underlying UV dynamics, and in particular the Higgs mass, which is out of control.
- #2: Reach the 1% goal for TH predictions for pp collisions by the start of the HL-LHC.
- #3: Develop MC tools that can provide all the necessary predictions/simulations within the planned computing and storage resources.
- #4: Bring the TH predictions for HTE factories to the level (0.1-0.01%) necessary for perspective studies.
- #5: Achieve understanding and precision predictions for SM (EW) phenomena at very high energy.



Data + computational complexity to be tamed. New technologies (ML, Quantum, GPU) to the rescue

Catch up with 20-year of TH developments after LEP

Cover physics of the SM at 100 TeV scales

Precision calculations for the HL-LHC

Goals	ETA wrt $t_{\text{HL-LHC}}$
$\text{NLO}_{\text{QCD}} \times \text{NLO}_{\text{EW}}$	Before
NNLO EW for $2 \rightarrow 2$	Well before
NNLO _{QCD} for (any) $2 \rightarrow 3$	Well before
PDF at N3LO	By
N3LO _{QCD} for main $2 \rightarrow 2$ (loops+subtraction)	Unknown
N4LO _{QCD} for candles	Unknown

Precision showering

Goals	ETA wrt to $t_{\text{HL-LHC}}$
NLL accuracy for QCD observables at pp collisions and implementation	Before
NNLO+NLL accuracy for QCD observables at hadron colliders	Before
NLO+LL with QCD+QED shower for any collider	Well before
NNLO+NLL accuracy with QCD+QED	At
LL EW Shower + Matching/merging	Well Before
NNLO+NLL Shower for lepton colliders	After